

4 Vergelijkingen en herleidingen

Voorkennis Stelsels lineaire vergelijkingen

Bladzijde 143

1

a

lijn l

x	0	2
y	-6	0

lijn m

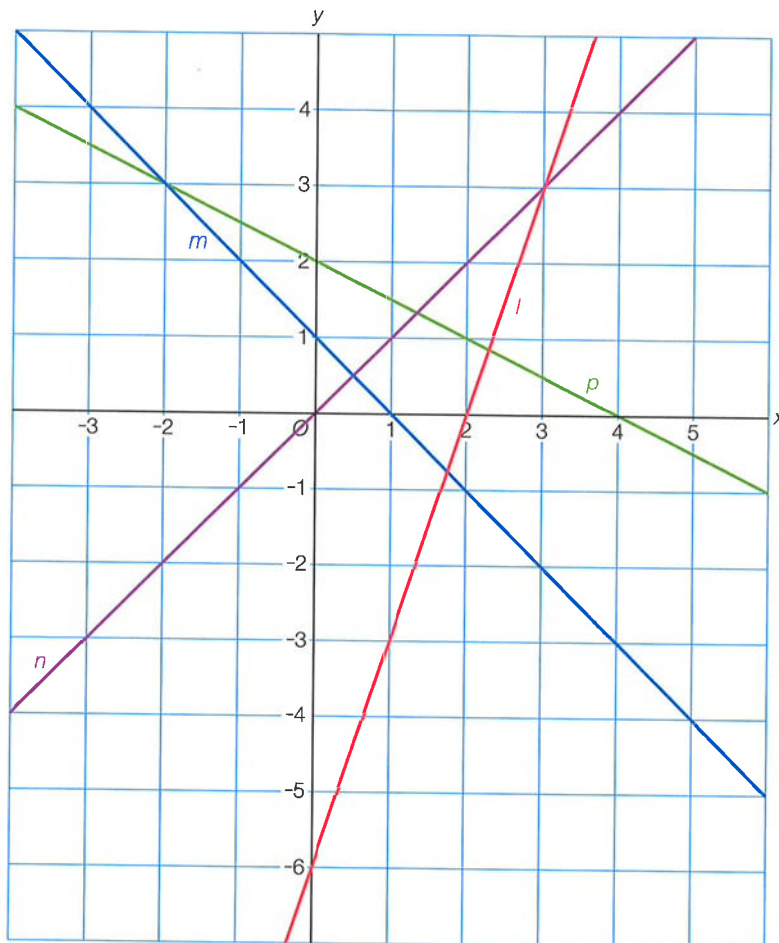
x	0	1
y	1	0

lijn n

x	0	2
y	0	2

lijn p

x	0	4
y	2	0



- b
- | | | | |
|------------------|-------------------|------------------|-----------------------------|
| $3x - y = 6$ | $x + y = 1$ | $x - y = 0$ | $x + 2y = 4$ |
| $-y = -3x + 6$ | $y = -x + 1$ | $-y = -x$ | $2y = -x + 4$ |
| $y = 3x - 6$ | Dus $rc_m = -1$. | $y = x$ | $y = -\frac{1}{2}x + 2$ |
| Dus $rc_l = 3$. | | Dus $rc_n = 1$. | Dus $rc_p = -\frac{1}{2}$. |

2

- a
- $y = 0$ invullen in $4x - 3y = 24$ geeft $4x = 24$ oftewel $x = 6$, dus het snijpunt met de x -as is $(6, 0)$.
- $x = 0$ invullen in $4x - 3y = 24$ geeft $-3y = 24$ oftewel $y = -8$, dus het snijpunt met de y -as is $(0, -8)$.
- b
- $x = 8$ en $y = 3$ invullen in $4x - 3y = 24$ geeft $\underbrace{4 \cdot 8 - 3 \cdot 3}_{=23} = 24$ en dat klopt niet, dus $A(8, 3)$ ligt niet op l .
- $x = 18$ en $y = 16$ invullen in $4x - 3y = 24$ geeft $\underbrace{4 \cdot 18 - 3 \cdot 16}_{=24} = 24$ en dat klopt, dus $B(18, 16)$ ligt op l .
- $x = -30$ en $y = -48$ invullen in $4x - 3y = 24$ geeft $\underbrace{4 \cdot -30 - 3 \cdot -48}_{=24} = 24$ en dat klopt, dus $C(-30, -48)$ ligt op l .

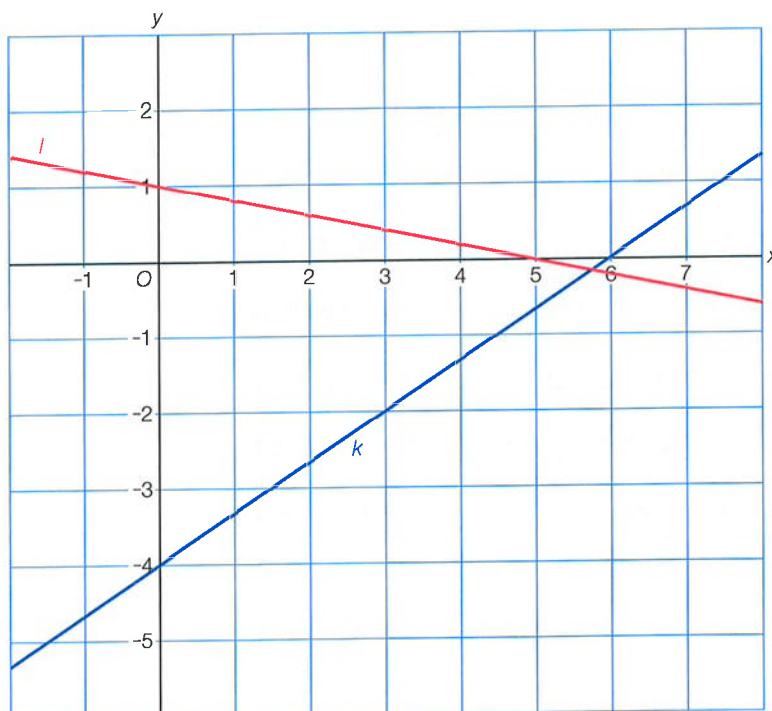
- c $x = 16$ en $y = p$ invullen in $4x - 3y = 24$ geeft $4 \cdot 16 - 3 \cdot p = 24$
 $64 - 3p = 24$
 $-3p = -40$
 $p = 13\frac{1}{3}$
- d $x = q$ en $y = 48$ invullen in $4x - 3y = 24$ geeft $4 \cdot q - 3 \cdot 48 = 24$
 $4q - 144 = 24$
 $4q = 168$
 $q = 42$

Dus voor $q = 42$.

Bladzijde 144

- 3 a Uit $x - 4y = 11$ volgt $x = 4y + 11$.
 $x = 4y + 11$ en $3x + y = 7$ geeft $3(4y + 11) + y = 7$
 $12y + 33 + y = 7$
 $13y = -26$
 $y = -2$
 $y = -2$ en $x = 4y + 11$ geeft $x = 4 \cdot -2 + 11 = 3$
 De oplossing is $(x, y) = (3, -2)$.
- b Uit $x + 2y = -3$ volgt $x = -2y - 3$.
 $x = -2y - 3$ en $4x - 3y = 10$ geeft $4(-2y - 3) - 3y = 10$
 $-8y - 12 - 3y = 10$
 $-11y = 22$
 $y = -2$
 $y = -2$ en $x = -2y - 3$ geeft $x = -2 \cdot -2 - 3 = 1$
 De oplossing is $(x, y) = (1, -2)$.
- c Uit $x - 6y = 1$ volgt $x = 6y + 1$.
 $x = 6y + 1$ en $5x + 4y = 56$ geeft $5(6y + 1) + 4y = 56$
 $30y + 5 + 4y = 56$
 $34y = 51$
 $y = 1\frac{1}{2}$
 $y = 1\frac{1}{2}$ en $x = 6y + 1$ geeft $x = 6 \cdot 1\frac{1}{2} + 1 = 10$
 De oplossing is $(x, y) = (10, 1\frac{1}{2})$.

- 4 a
- | lijn k | | |
|--------|----|---|
| x | 0 | 6 |
| y | -4 | 0 |
- | lijn l | | |
|--------|---|---|
| x | 0 | 5 |
| y | 1 | 0 |



- b Uit $x + 5y = 5$ volgt $x = -5y + 5$.
 $x = -5y + 5$ en $2x - 3y = 12$ geeft $2(-5y + 5) - 3y = 12$
 $-10y + 10 - 3y = 12$
 $-13y = 2$
 $y = -\frac{2}{13}$
 $y = -\frac{2}{13}$ en $x = -5y + 5$ geeft $x = -5 \cdot -\frac{2}{13} + 5 = 5\frac{10}{13}$
Dus $S(5\frac{10}{13}, -\frac{2}{13})$.

4.1 Stelsels vergelijkingen

Bladzijde 145

- 1 x vrijmaken bij $7x + 4y = 1$ geeft $7x + 4y = 1$
 $7x = 1 - 4y$
 $x = \frac{1}{7} - \frac{4}{7}y$
Substitutie van $x = \frac{1}{7} - \frac{4}{7}y$ in $5x - 4y = 11$ geeft $5(\frac{1}{7} - \frac{4}{7}y) - 4y = 11$
 $\frac{5}{7} - 2\frac{6}{7}y - 4y = 11$
 $5 - 20y - 28y = 77$
 $-48y = 72$
 $y = -1\frac{1}{2}$
 $y = -1\frac{1}{2}$ geeft $x = \frac{1}{7} - \frac{4}{7} \cdot -1\frac{1}{2} = 1$
De oplossing is $(x, y) = (1, -1\frac{1}{2})$.

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- 2 a $\begin{cases} 5x - 4y = -8 \\ -x + 4y = -12 \end{cases} +$
 $\frac{4x}{4x} = -20 +$
 $\begin{cases} x = -5 \\ -x + 4y = -12 \end{cases} \begin{cases} 5 + 4y = -12 \\ 4y = -17 \end{cases}$
 $y = -4\frac{1}{4}$
De oplossing is $(x, y) = (-5, -4\frac{1}{4})$.
b $\begin{cases} -2x + y = 7 \\ -2x + 3y = -1 \end{cases} -$
 $\frac{-2y}{-2y} = 8 -$
 $\begin{cases} y = -4 \\ -2x + y = 7 \end{cases} \begin{cases} -2x - 4 = 7 \\ -2x = 11 \end{cases}$
 $x = -5\frac{1}{2}$
De oplossing is $(x, y) = (-5\frac{1}{2}, -4)$.
c $\begin{cases} -x - 3y = -8 \\ -2x + 3y = -1 \end{cases} +$
 $\frac{-3x}{-3x} = -9 +$
 $\begin{cases} x = 3 \\ -2x + 3y = -1 \end{cases} \begin{cases} -6 + 3y = -1 \\ 3y = 5 \end{cases}$
 $y = 1\frac{2}{3}$
De oplossing is $(x, y) = (3, 1\frac{2}{3})$.

Bladzijde 147

- 3 Stel van elke driehoek de lengte van de basis x en de lengte van de benen y .
Omtrek driehoek is 14 geeft $x + 2y = 14$.
Omtrek figuur is 30 geeft $5x + 2y = 30$.
 $\begin{cases} x + 2y = 14 \\ 5x + 2y = 30 \end{cases} -$
 $\frac{-4x}{-4x} = -16 -$
 $\begin{cases} x = 4 \\ x + 2y = 14 \end{cases} \begin{cases} 4 + 2y = 14 \\ 2y = 10 \end{cases}$
 $y = 5$
De zijden zijn 4 (basis) en 5 (benen).

$$\begin{array}{lcl}
 \text{4} & \begin{cases} a+b+c=20 \\ a+b+d=24 \\ \hline c-d=-4 \\ c=d-4 \end{cases} & \begin{cases} a+b+c=20 \\ a+c+d=25 \\ \hline b-d=-5 \\ b=d-5 \end{cases} & \begin{cases} a+b+c=20 \\ b+c+d=27 \\ \hline a-d=-7 \\ a=d-7 \end{cases}
 \end{array}$$

Substitutie van $c = d - 4$, $b = d - 5$ en $a = d - 7$ in $a + b + c = 20$ geeft

$$d - 7 + d - 5 + d - 4 = 20$$

$$3d - 16 = 20$$

$$3d = 36$$

$$d = 12$$

$d = 12$ geeft $a = d - 7 = 12 - 7 = 5$, $b = d - 5 = 12 - 5 = 7$ en $c = d - 4 = 12 - 4 = 8$.

De oplossing is $(a, b, c, d) = (5, 7, 8, 12)$.

$$\text{5 a } \begin{cases} 3x - 4y = 7 \\ 2x + 3y = 16 \\ \hline 5x - y = 23 \end{cases} +$$

Er is geen variabele geëlimineerd.

$$\text{b } \begin{cases} 3x - 4y = 7 \\ 2x + 3y = 16 \\ \hline x - 7y = -9 \end{cases} -$$

Er is geen variabele geëlimineerd.

- c Als je linker- en rechterlid van de bovenste vergelijking met 3 vermenigvuldigt en van de onderste vergelijking met 4, dan krijg je een stelsel waarmee je wel door optellen een variabele elimineert.

$$\begin{cases} 3x - 4y = 7 \\ 2x + 3y = 16 \end{cases} \begin{array}{l} 3 \\ 4 \end{array} \text{ geeft } \begin{cases} 9x - 12y = 21 \\ 8x + 12y = 64 \end{cases}$$

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$$\text{6 a } \begin{cases} 3x + 5y = -7 \\ 2x + y = 0 \end{cases} \begin{array}{l} 1 \\ 5 \end{array} \text{ geeft } \begin{cases} 3x + 5y = -7 \\ 10x + 5y = 0 \\ \hline -7x = -7 \end{cases}$$

$$\begin{aligned}
 x &= 1 \\
 2x + y &= 0 \\
 y &= -2
 \end{aligned}$$

De oplossing is $(x, y) = (1, -2)$.

$$\text{b } \begin{cases} x - 2y = 3 \\ 3x - y = 19 \end{cases} \begin{array}{l} 1 \\ 2 \end{array} \text{ geeft } \begin{cases} x - 2y = 3 \\ 6x - 2y = 38 \\ \hline -5x = -35 \end{cases}$$

$$\begin{aligned}
 x &= 7 \\
 3x - y &= 19 \\
 y &= 2
 \end{aligned}$$

De oplossing is $(x, y) = (7, 2)$.

$$\text{c } \begin{cases} 4x + y = 13 \\ x - 2y = 1 \end{cases} \begin{array}{l} 2 \\ 1 \end{array} \text{ geeft } \begin{cases} 8x + 2y = 26 \\ x - 2y = 1 \\ \hline 9x = 27 \end{cases}$$

$$\begin{aligned}
 x &= 3 \\
 4x + y &= 13 \\
 y &= 1
 \end{aligned}$$

De oplossing is $(x, y) = (3, 1)$.

$$\text{7 a } \begin{cases} 5x + 2y = 69 \\ x + 3y = -7 \end{cases} \begin{array}{l} 1 \\ 5 \end{array} \text{ geeft } \begin{cases} 5x + 2y = 69 \\ 5x + 15y = -35 \\ \hline -13y = 104 \end{cases}$$

$$\begin{aligned}
 y &= -8 \\
 x + 3y &= -7 \\
 x &= 17
 \end{aligned}$$

De oplossing is $(x, y) = (17, -8)$.

$$\text{b } \begin{cases} 2x - 5y = -19 \\ 5x + 4y = 35 \end{cases} \begin{vmatrix} 4 \\ 5 \end{vmatrix} \text{ geeft } \begin{cases} 8x - 20y = -76 \\ 25x + 20y = 175 \end{cases} +$$

$$\begin{array}{r} 33x \\ x = 3 \\ 2x - 5y = -19 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} 2 \cdot 3 - 5y = -19 \\ 6 - 5y = -19 \\ -5y = -25 \\ y = 5 \end{array}$$

De oplossing is $(x, y) = (3, 5)$.

$$\text{c } \begin{cases} 0,8x + 0,2y = 1 \\ 0,3x - 0,3y = 1,5 \end{cases} \begin{vmatrix} 3 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} 2,4x + 0,6y = 3 \\ 0,6x - 0,6y = 3 \end{cases} +$$

$$\begin{array}{r} 3x \\ x = 2 \\ 0,8x + 0,2y = 1 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} 0,8 \cdot 2 + 0,2y = 1 \\ 1,6 + 0,2y = 1 \\ 0,2y = -0,6 \\ y = -3 \end{array}$$

De oplossing is $(x, y) = (2, -3)$.

$$\text{8 a } \begin{cases} 5x - y = 3\sqrt{2} \\ 3x + 2y = 7\sqrt{2} \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix} \text{ geeft } \begin{cases} 10x - 2y = 6\sqrt{2} \\ 3x + 2y = 7\sqrt{2} \end{cases} +$$

$$\begin{array}{r} 13x \\ x = \sqrt{2} \\ 3x + 2y = 7\sqrt{2} \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} 3\sqrt{2} + 2y = 7\sqrt{2} \\ 2y = 4\sqrt{2} \\ y = 2\sqrt{2} \end{array}$$

De oplossing is $(x, y) = (\sqrt{2}, 2\sqrt{2})$.

$$\text{b } \begin{cases} x\sqrt{2} + y = \sqrt{2} \\ x + y\sqrt{2} = 1 + \sqrt{2} \end{cases} \begin{vmatrix} \sqrt{2} \\ 1 \end{vmatrix} \text{ geeft } \begin{cases} 2x + y\sqrt{2} = 2 \\ x + y\sqrt{2} = 1 + \sqrt{2} \end{cases} -$$

$$\begin{array}{r} x = 1 - \sqrt{2} \\ x + y\sqrt{2} = 1 + \sqrt{2} \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} 1 - \sqrt{2} + y\sqrt{2} = 1 + \sqrt{2} \\ y\sqrt{2} = 2\sqrt{2} \\ y = 2 \end{array}$$

De oplossing is $(x, y) = (1 - \sqrt{2}, 2)$.

$$\text{c } \begin{cases} 4x\sqrt{3} - 5y\sqrt{2} = 2\sqrt{6} \\ x\sqrt{2} + y\sqrt{3} = 12 \end{cases} \begin{vmatrix} \sqrt{2} \\ 4\sqrt{3} \end{vmatrix} \text{ geeft } \begin{cases} 4x\sqrt{6} - 10y = 4\sqrt{3} \\ 4x\sqrt{6} + 12y = 48\sqrt{3} \end{cases} -$$

$$\begin{array}{r} -22y = -44\sqrt{3} \\ y = 2\sqrt{3} \\ x\sqrt{2} + y\sqrt{3} = 12 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} x\sqrt{2} + 6 = 12 \\ x\sqrt{2} = 6 \\ x = \frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2} \end{array}$$

De oplossing is $(x, y) = (3\sqrt{2}, 2\sqrt{3})$.

$$\text{9 a } \begin{cases} 3x - 2y = -12 \\ x + 4y = 38 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix} \text{ geeft } \begin{cases} 6x - 4y = -24 \\ x + 4y = 38 \end{cases} +$$

$$\begin{array}{r} 7x \\ x = 2 \\ x + 4y = 38 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} 2 + 4y = 38 \\ 4y = 36 \\ y = 9 \end{array}$$

Het snijpunt is $(2, 9)$.

$$\begin{aligned} \text{b } \begin{cases} 2x + 5y = 26 \\ 3x - 2y = 1 \end{cases} \begin{vmatrix} 2 \\ 5 \end{vmatrix} \text{ geeft } \begin{cases} 4x + 10y = 52 \\ 15x - 10y = 5 \end{cases} + \\ \hline 19x = 57 \\ x = 3 \\ 2x + 5y = 26 \end{cases} \begin{cases} 2 \cdot 3 + 5y = 26 \\ 6 + 5y = 26 \\ 5y = 20 \\ y = 4 \end{cases} \end{aligned}$$

Het snijpunt is (3, 4).

- 10 a** Telkens worden de laatste twee getallen bij elkaar opgeteld om het volgende getal te krijgen.

Het zevende getal is dus $2a + 3b + 3a + 5b = 5a + 8b$.

Het achtste getal is $3a + 5b + 5a + 8b = 8a + 13b$.

$$\begin{aligned} \text{b } \begin{cases} 5a + 8b = 34 \\ 8a + 13b = 55 \end{cases} \begin{vmatrix} 8 \\ 5 \end{vmatrix} \text{ geeft } \begin{cases} 40a + 64b = 272 \\ 40a + 65b = 275 \end{cases} - \\ \hline -b = -3 \\ b = 3 \\ 5a + 8b = 34 \end{cases} \begin{cases} 5a + 8 \cdot 3 = 34 \\ 5a + 24 = 34 \\ 5a = 10 \\ a = 2 \end{cases} \end{aligned}$$

Dus $a = 2$ en $b = 3$.

$$\begin{aligned} \text{11 } \begin{cases} ax + by = c \\ px + qy = r \end{cases} \begin{vmatrix} q \\ b \end{vmatrix} \text{ geeft } \begin{cases} aqx + bqy = cq \\ bpx + bqy = br \end{cases} - \\ \hline aqx - bpx = cq - br \\ (aq - bp)x = cq - br \\ x = \frac{cq - br}{aq - bp} \end{aligned}$$

$$\text{Dus } x = \frac{cq - br}{aq - bp}.$$

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- 12** Stel in het bad kan T liter water.

Stel uit de kranen A en B stroomt per minuut $a + b$ liter water, dan geldt $a + b = \frac{1}{4}T$.

Stel uit de kranen A en C stroomt per minuut $a + c$ liter water, dan geldt $a + c = \frac{1}{5}T$.

Stel uit de kranen B en C stroomt per minuut $b + c$ liter water, dan geldt $b + c = \frac{1}{6}T$.

$$a + b + a + c + b + c = \frac{1}{4}T + \frac{1}{5}T + \frac{1}{6}T \text{ liter per minuut}$$

$$2a + 2b + 2c = \frac{37}{60}T \text{ liter per minuut}$$

$$a + b + c = \frac{37}{120}T \text{ liter per minuut}$$

Dus als alle drie de kranen tegelijk worden opengedraaid, is het bad vol in $\frac{120}{37}$ oftewel $3\frac{9}{37}$ minuten.

- 13 a** $x = 1$ en $y = -2$ invullen in $y = x^2 + bx + c$ geeft $-2 = 1^2 + b + c$, dus $b + c = -3$.

- b** $x = 2$ en $y = 3$ invullen in $y = x^2 + bx + c$ geeft $3 = 2^2 + 2b + c$, dus $2b + c = -1$.

$$\begin{aligned} \text{c } \begin{cases} b + c = -3 \\ 2b + c = -1 \end{cases} - \\ \hline -b = -2 \\ b = 2 \\ b + c = -3 \end{cases} \begin{cases} 2 + c = -3 \\ c = -5 \end{cases} \end{aligned}$$

Dus $b = 2$ en $c = -5$.

14 a (1, 5) op $y = ax^2 + bx$ geeft $a + b = 5$.
 (2, 14) op $y = ax^2 + bx$ geeft $4a + 2b = 14$ oftewel $2a + b = 7$.

$$\begin{cases} a + b = 5 \\ 2a + b = 7 \end{cases}$$

$$\begin{array}{r} a + b = 5 \\ -a = -2 \\ \hline a = 2 \end{array}$$

$$\begin{cases} a = 2 \\ a + b = 5 \end{cases} \Rightarrow \begin{cases} 2 + b = 5 \\ b = 3 \end{cases}$$

De formule is $y = 2x^2 + 3x$.

b (1, 8) op $y = ax^2 + c$ geeft $a + c = 8$.
 (2, 17) op $y = ax^2 + c$ geeft $4a + c = 17$.

$$\begin{cases} a + c = 8 \\ 4a + c = 17 \end{cases}$$

$$\begin{array}{r} a + c = 8 \\ -3a = -9 \\ \hline a = 3 \end{array}$$

$$\begin{cases} a = 3 \\ a + c = 8 \end{cases} \Rightarrow \begin{cases} 3 + c = 8 \\ c = 5 \end{cases}$$

De formule is $y = 3x^2 + 5$.

15 a (2, -1) op $y = x^2 + px + q$ geeft $4 + 2p + q = -1$ oftewel $2p + q = -5$.
 (2, -1) op $y = 2px - q$ geeft $4p - q = -1$.

$$\begin{cases} 2p + q = -5 \\ 4p - q = -1 \end{cases}$$

$$\begin{array}{r} 2p + q = -5 \\ 4p - q = -1 \\ \hline 6p = -6 \end{array}$$

$$\begin{cases} p = -1 \\ 2p + q = -5 \end{cases} \Rightarrow \begin{cases} -2 + q = -5 \\ q = -3 \end{cases}$$

b $p = -1$ en $q = -3$ geeft $y = x^2 - x - 3$ en $y = -2x + 3$.

$$\begin{aligned} x^2 - x - 3 &= -2x + 3 \\ x^2 + x - 6 &= 0 \\ (x - 2)(x + 3) &= 0 \\ x &= 2 \vee x = -3 \\ x = -3 &\text{ geeft } y = -2 \cdot -3 + 3 = 9 \end{aligned}$$

Het andere snijpunt is (-3, 9).

16 (0, 4) op $y = ax^2 + bx + c$ geeft $0 + 0 + c = 4$, dus $c = 4$.
 (-2, -10) op $y = ax^2 + bx + 4$ geeft $4a - 2b + 4 = -10$ oftewel $4a - 2b = -14$, dus $2a - b = -7$.
 (3, 5) op $y = ax^2 + bx + 4$ geeft $9a + 3b + 4 = 5$ oftewel $9a + 3b = 1$.

$$\begin{cases} 2a - b = -7 \\ 9a + 3b = 1 \end{cases} \begin{array}{l} \left| \begin{array}{l} 3 \\ 1 \end{array} \right| \text{ geeft } \begin{cases} 6a - 3b = -21 \\ 9a + 3b = 1 \end{cases} \\ \hline 15a = -20 \\ a = -1\frac{1}{3} \end{array}$$

$$\begin{cases} a = -1\frac{1}{3} \\ 2a - b = -7 \end{cases} \Rightarrow \begin{cases} 2 \cdot -1\frac{1}{3} - b = -7 \\ -2\frac{2}{3} - b = -7 \\ -b = -4\frac{1}{3} \\ b = 4\frac{1}{3} \end{cases}$$

De formule is $y = -1\frac{1}{3}x^2 + 4\frac{1}{3}x + 4$.

17 $f(x) = ax^2 + bx + c$ geeft $f'(x) = 2ax + b$
 $f'(2) = rc_k$ geeft $4a + b = 1$.
 $f'(8) = rc_l$ geeft $16a + b = -2$.

$$\begin{cases} 4a + b = 1 \\ 16a + b = -2 \end{cases}$$

$$\begin{array}{r} 4a + b = 1 \\ -12a = 3 \\ \hline -12a = 3 \end{array}$$

$$\begin{cases} a = -\frac{1}{4} \\ 4a + b = 1 \end{cases} \Rightarrow \begin{cases} 4 \cdot -\frac{1}{4} + b = 1 \\ -1 + b = 1 \\ b = 2 \end{cases}$$

$a = -\frac{1}{4}$ en $b = 2$ geeft $f(x) = -\frac{1}{4}x^2 + 2x + c$.

$$f(2) = 6 \text{ geeft } -\frac{1}{4} \cdot 2^2 + 2 \cdot 2 + c = 6$$

$$-1 + 4 + c = 6$$

$$c = 3$$

Dit geeft $f(x) = -\frac{1}{4}x^2 + 2x + 3$.

$$f(8) = -\frac{1}{4} \cdot 8^2 + 2 \cdot 8 + 3 = 3, \text{ dus } (8, 3) \text{ ook op de grafiek van } f.$$

Dus $a = -\frac{1}{4}$, $b = 2$ en $c = 3$.

- 18 a** Je kunt de vergelijking $x + y = 3$ niet met een getal vermenigvuldigen zodat na optellen of aftrekken van de vergelijking $x^2 + y^2 = 5$ een variabele wordt geëlimineerd. Ook kun je de vergelijking $x^2 + y^2 = 5$ niet met een getal vermenigvuldigen zodat na optellen of aftrekken van de vergelijking $x + y = 3$ een variabele wordt geëlimineerd. Dus je kunt het gegeven stelsel niet oplossen met behulp van elimineren door optellen of aftrekken.
- b** $x = 1$ en $y = 2$ invullen in $x + y = 3$ geeft $1 + 2 = 3$ en dat klopt.
 $x = 1$ en $y = 2$ invullen in $x^2 + y^2 = 5$ geeft $1 + 4 = 5$ en dat klopt.
 Dus $(x, y) = (1, 2)$ is een oplossing van het stelsel.
 $x = 2$ en $y = 1$ invullen in $x + y = 3$ geeft $2 + 1 = 3$ en dat klopt.
 $x = 2$ en $y = 1$ invullen in $x^2 + y^2 = 5$ geeft $4 + 1 = 5$ en dat klopt.
 Dus $(x, y) = (2, 1)$ is een oplossing van het stelsel.

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- 19 a** Substitutie van $y = x^2 - 18$ in $x + y = 2$ geeft $x + x^2 - 18 = 2$
- $$x^2 + x - 20 = 0$$
- $$(x - 4)(x + 5) = 0$$
- $$x = 4 \vee x = -5$$
- $x = 4$ invullen in $y = x^2 - 18$ geeft $y = 4^2 - 18 = -2$.
 $x = -5$ invullen in $y = x^2 - 18$ geeft $y = (-5)^2 - 18 = 7$.
 Dus $(x, y) = (4, -2) \vee (x, y) = (-5, 7)$.
- b** Substitutie van $y = x^2 - 3$ in $x - y = -3$ geeft $x - (x^2 - 3) = -3$
- $$x - x^2 + 3 = -3$$
- $$-x^2 + x + 6 = 0$$
- $$x^2 - x - 6 = 0$$
- $$(x + 2)(x - 3) = 0$$
- $$x = -2 \vee x = 3$$
- $x = -2$ invullen in $y = x^2 - 3$ geeft $y = (-2)^2 - 3 = 1$.
 $x = 3$ invullen in $y = x^2 - 3$ geeft $y = 3^2 - 3 = 6$.
 Dus $(x, y) = (-2, 1) \vee (x, y) = (3, 6)$.
- c** $3x + y = 5$ oftewel $y = 5 - 3x$
 Substitutie van $y = 5 - 3x$ in $x^2 + y^2 = 25$ geeft $x^2 + (5 - 3x)^2 = 25$
- $$x^2 + 25 - 30x + 9x^2 = 25$$
- $$10x^2 - 30x = 0$$
- $$10x(x - 3) = 0$$
- $$x = 0 \vee x = 3$$
- $x = 0$ invullen in $y = 5 - 3x$ geeft $y = 5 - 0 = 5$.
 $x = 3$ invullen in $y = 5 - 3x$ geeft $y = 5 - 9 = -4$.
 Dus $(x, y) = (0, 5) \vee (x, y) = (3, -4)$.

- 20 a** $2x + y = 4$ oftewel $y = 4 - 2x$
 Substitutie van $y = 4 - 2x$ in $(x - 3)^2 + y^2 = 8$ geeft $(x - 3)^2 + (4 - 2x)^2 = 8$
- $$x^2 - 6x + 9 + 16 - 16x + 4x^2 = 8$$
- $$5x^2 - 22x + 17 = 0$$
- $$D = (-22)^2 - 4 \cdot 5 \cdot 17 = 144$$
- $$x = \frac{22 \pm 12}{10} = 3\frac{2}{5} \vee x = \frac{22 - 12}{10} = 1$$
- $x = 3\frac{2}{5}$ invullen in $y = 4 - 2x$ geeft $y = 4 - 2 \cdot 3\frac{2}{5} = -2\frac{4}{5}$.
 $x = 1$ invullen in $y = 4 - 2x$ geeft $y = 4 - 2 = 2$.
 Dus $(x, y) = (3\frac{2}{5}, -2\frac{4}{5}) \vee (x, y) = (1, 2)$.

- b** $x + y = 9$ oftewel $y = 9 - x$

$$\begin{aligned}\text{Substitutie van } y = 9 - x \text{ in } xy = 20 \text{ geeft } x(9 - x) &= 20 \\ 9x - x^2 &= 20 \\ x^2 - 9x + 20 &= 0 \\ (x - 4)(x - 5) &= 0 \\ x &= 4 \vee x = 5\end{aligned}$$

$x = 4$ invullen in $y = 9 - x$ geeft $y = 9 - 4 = 5$.

$x = 5$ invullen in $y = 9 - x$ geeft $y = 9 - 5 = 4$.

Dus $(x, y) = (4, 5) \vee (x, y) = (5, 4)$.

- c** $4x + y = 16$ oftewel $y = 16 - 4x$

$$\begin{aligned}\text{Substitutie van } y = 16 - 4x \text{ in } xy = 15 \text{ geeft } x(16 - 4x) &= 15 \\ 16x - 4x^2 &= 15 \\ 4x^2 - 16x + 15 &= 0 \\ D &= (-16)^2 - 4 \cdot 4 \cdot 15 = 16 \\ x &= \frac{16 \pm 4}{8} = 2\frac{1}{2} \vee x = \frac{16 - 4}{8} = 1\frac{1}{2}\end{aligned}$$

$x = 1\frac{1}{2}$ invullen in $y = 16 - 4x$ geeft $y = 16 - 6 = 10$.

$x = 2\frac{1}{2}$ invullen in $y = 16 - 4x$ geeft $y = 16 - 10 = 6$.

Dus $(x, y) = (1\frac{1}{2}, 10) \vee (x, y) = (2\frac{1}{2}, 6)$.

- 21** Zie de figuur hiernaast met ruit $ABCD$.
 S is het snijpunt de diagonalen AC en BD .

Stel $AS = x$ en $BS = y$.

In een ruit delen de diagonalen elkaar middendoor, dus $AC + BD = 2x + 2y$.

$$2x + 2y = 34 \text{ oftewel } x + y = 17$$

In een ruit staan de diagonalen loodrecht op elkaar, dus $\angle ASB = 90^\circ$

$$\text{en } O(ABCD) = 4 \cdot O(\triangle ABS) = 4 \cdot \frac{1}{2}xy = 2xy.$$

$$2xy = 120 \text{ oftewel } xy = 60$$

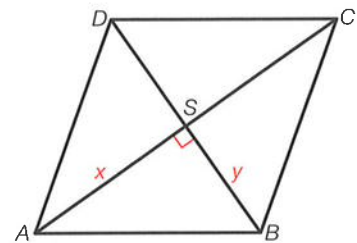
$$x + y = 17 \text{ geeft } y = 17 - x$$

$$\text{Substitutie van } y = 17 - x \text{ in } xy = 60 \text{ geeft } x(17 - x) = 60$$

$$\begin{aligned}17x - x^2 &= 60 \\ x^2 - 17x + 60 &= 0 \\ (x - 5)(x - 12) &= 0 \\ x &= 5 \vee x = 12\end{aligned}$$

$$x = 5 \text{ geeft } y = 17 - 5 = 12 \text{ en } x = 12 \text{ geeft } y = 17 - 12 = 5$$

Dus de lengte van de zijde van de ruit is $\sqrt{12^2 + 5^2} = 13$.



- 22** Zie de figuur hiernaast.

$ABCD$ is gelijkbenig met $AD = BC$, dus FG is symmetrieas.

M is het middelpunt van de cirkel die door A, B, C en D gaat.

Stel $AM = BM = CM = DM = r$.

De stelling van Pythagoras in $\triangle ADE$ geeft $DE = \sqrt{4^2 - 1^2} = \sqrt{15}$.

$$FG = DE = \sqrt{15}$$

Stel $FM = x$, dan is $GM = \sqrt{15} - x$.

De stelling van Pythagoras in $\triangle AFM$ geeft $AM^2 = AF^2 + FM^2$

$$\begin{aligned}r^2 &= 2^2 + x^2 \\ r^2 &= 4 + x^2\end{aligned}$$

De stelling van Pythagoras in $\triangle DGM$ geeft $DM^2 = DG^2 + GM^2$

$$\begin{aligned}r^2 &= 1^2 + (\sqrt{15} - x)^2 \\ r^2 &= 1 + 15 - 2x\sqrt{15} + x^2 \\ r^2 &= 16 - 2x\sqrt{15} + x^2\end{aligned}$$

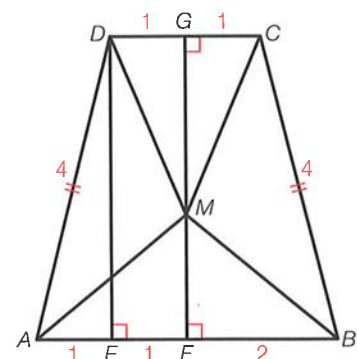
$$\text{Dus } 4 + x^2 = 16 - 2x\sqrt{15} + x^2$$

$$2x\sqrt{15} = 12$$

$$x = \frac{12}{2\sqrt{15}} = \frac{6}{\sqrt{15}}$$

$$x = \frac{6}{\sqrt{15}} \text{ geeft } r^2 = 4 + \left(\frac{6}{\sqrt{15}}\right)^2 = 4 + \frac{36}{15} = \frac{32}{5}$$

De oppervlakte van de cirkel die door de punten A, B, C en D gaat, is $\pi \cdot \frac{32}{5} = 6\frac{2}{5}\pi$.



4.2 Hogeregraadsvergelijkingen

Bladzijde 152

- 23 a** De vergelijking $x^3 = 10$ heeft één oplossing.
De vergelijking $x^3 = -10$ heeft één oplossing.
- b** De vergelijking $x^4 = 10$ heeft twee oplossingen.
De vergelijking $x^4 = -10$ heeft nul oplossingen.

Bladzijde 153

24 a

x	x^2	x^3	x^4	x^5	x^6
1	1	1	1	1	1
2	4	8	16	32	64
3	9	27	81	243	729
4	16	64	256	1024	
5	25	125	625		
6	36	216			

b *

Bladzijde 154

25 a $5x^3 = 135$
 $x^3 = 27$
 $x = 3$

b $1 - 3x^5 = 97$
 $-3x^5 = 96$
 $x^5 = -32$
 $x = -2$

c $\frac{1}{4}x^8 + 3 = 10$
 $\frac{1}{4}x^8 = 7$
 $x^8 = 28$
 $x = \sqrt[8]{28} \vee x = -\sqrt[8]{28}$

d $5x^3 - 1 = 9$
 $5x^3 = 10$
 $x^3 = 2$
 $x = \sqrt[3]{2}$

e $8x^3 + 2 = 1$
 $8x^3 = -1$
 $x^3 = -\frac{1}{8}$
 $x = -\frac{1}{2}$

f $5x^6 + 7 = 97$
 $5x^6 = 90$
 $x^6 = 18$
 $x = \sqrt[6]{18} \vee x = -\sqrt[6]{18}$

26 a $5x^4 - 3 = 17$
 $5x^4 = 20$
 $x^4 = 4$
 $x = \sqrt[4]{4} \vee x = -\sqrt[4]{4}$

b $4x^3 - 5 = 1367$
 $4x^3 = 1372$
 $x^3 = 343$
 $x = 7$

c $3(4x - 5)^3 = 15$
 $(4x - 5)^3 = 5$
 $4x - 5 = \sqrt[3]{5}$
 $4x = 5 + \sqrt[3]{5}$
 $x = 1\frac{1}{4} + \frac{1}{4} \cdot \sqrt[3]{5}$

d $13 - 2(1 - x)^4 = 1$
 $-2(1 - x)^4 = -12$
 $(1 - x)^4 = 6$
 $1 - x = \sqrt[4]{6} \vee 1 - x = -\sqrt[4]{6}$
 $-x = -1 + \sqrt[4]{6} \vee -x = -1 - \sqrt[4]{6}$
 $x = 1 - \sqrt[4]{6} \vee x = 1 + \sqrt[4]{6}$

27 a $\frac{1}{4}x^3 + 60 = 6$
 $\frac{1}{4}x^3 = -54$
 $x^3 = -216$
 $x = -6$

b $100 - 3x^4 = 55$
 $-3x^4 = -45$
 $x^4 = 15$
 $x = \sqrt[4]{15} \vee x = -\sqrt[4]{15}$

c $\frac{1}{2}(4x - 1)^5 + 3 = 19$
 $\frac{1}{2}(4x - 1)^5 = 16$
 $(4x - 1)^5 = 32$
 $4x - 1 = 2$
 $4x = 3$
 $x = \frac{3}{4}$

d $2\frac{1}{2}(1 - 2x)^6 - 6 = 14$
 $2\frac{1}{2}(1 - 2x)^6 = 20$
 $(1 - 2x)^6 = 8$
 $1 - 2x = \sqrt[6]{8} \vee 1 - 2x = -\sqrt[6]{8}$
 $-2x = -1 + \sqrt[6]{8} \vee -2x = -1 - \sqrt[6]{8}$
 $x = \frac{1}{2} - \frac{1}{2} \cdot \sqrt[6]{8} \vee x = \frac{1}{2} + \frac{1}{2} \cdot \sqrt[6]{8}$

28 $f(x) = px^4 + 3x^2$ geeft $f'(x) = 4px^3 + 6x$
 rc = 0, dus $f'(x) = 0$ geeft $4px^3 + 6x = 0$
 $2x(2px^2 + 3) = 0$
 $x = 0 \vee 2px^2 = -3$
 $x = 0 \vee x^2 = -\frac{3}{2p}$
 $x = 0 \vee x = \sqrt{-\frac{3}{2p}} \vee x = -\sqrt{-\frac{3}{2p}}$

Dus $x_A = -\sqrt{-\frac{3}{2p}}$ en $x_B = \sqrt{-\frac{3}{2p}}$.

29 a $2x^4 + 2a = a^2 + 4$
 $2x^4 = a^2 - 2a + 4$
 $x^4 = \frac{1}{2}a^2 - a + 2$
 $x = \sqrt[4]{\frac{1}{2}a^2 - a + 2} \vee x = -\sqrt[4]{\frac{1}{2}a^2 - a + 2}$

b $2x^4 + 2a = a^2 + 4$
 $a^2 - 2a + 4 - 2x^4 = 0$
 $(a-1)^2 - 1 + 4 - 2x^4 = 0$
 $(a-1)^2 = 2x^4 - 3$
 $a-1 = \sqrt{2x^4 - 3} \vee a-1 = -\sqrt{2x^4 - 3}$
 $a = 1 + \sqrt{2x^4 - 3} \vee a = 1 - \sqrt{2x^4 - 3}$

30 a $x^3 - x^2 - 6x = x(x^2 - x - 6) = x(x+2)(x-3)$
b $x = 0 \vee x = -2 \vee x = 3$

31 a Het linker- en rechterlid van $x^2 = u$ kwadrateren geeft $(x^2)^2 = u^2$ oftewel $x^4 = u^2$.
 Substitutie van $x^4 = u^2$ en $x^2 = u$ in $x^4 - x^2 - 6 = 0$ geeft $u^2 - u - 6 = 0$

b $u^2 - u - 6 = 0$
 $(u+2)(u-3) = 0$
 $u = -2 \vee u = 3$

c $\left. \begin{array}{l} u = -2 \\ x^2 = u \end{array} \right\} x^2 = -2$ geen oplossingen
 $\left. \begin{array}{l} u = 3 \\ x^2 = u \end{array} \right\} x^2 = 3$
 $x = \sqrt{3} \vee x = -\sqrt{3}$

Bladzijde 155

32 a $x^3 - 5x^2 + 6x = 0$
 $x(x^2 - 5x + 6) = 0$
 $x(x-2)(x-3) = 0$
 $x = 0 \vee x = 2 \vee x = 3$

b $x^3 - 5x^2 = 6x$
 $x^3 - 5x^2 - 6x = 0$
 $x(x^2 - 5x - 6) = 0$
 $x(x+1)(x-6) = 0$
 $x = 0 \vee x = -1 \vee x = 6$

c $x^3 = 4x^2 + 12x$
 $x^3 - 4x^2 - 12x = 0$
 $x(x^2 - 4x - 12) = 0$
 $x(x+2)(x-6) = 0$
 $x = 0 \vee x = -2 \vee x = 6$

d $x^4 - 13x^2 + 36 = 0$
 Stel $x^2 = u$.
 $u^2 - 13u + 36 = 0$
 $(u-4)(u-9) = 0$
 $u = 4 \vee u = 9$
 $x^2 = 4 \vee x^2 = 9$
 $x = 2 \vee x = -2 \vee x = 3 \vee x = -3$

33 a $x^4 - 10x^2 + 9 = 0$
 Stel $x^2 = u$.
 $u^2 - 10u + 9 = 0$
 $(u-1)(u-9) = 0$
 $u = 1 \vee u = 9$
 $x^2 = 1 \vee x^2 = 9$
 $x = 1 \vee x = -1 \vee x = 3 \vee x = -3$

b $x^4 - 8x^2 - 9 = 0$
 Stel $x^2 = u$.
 $u^2 - 8u - 9 = 0$
 $(u+1)(u-9) = 0$
 $u = -1 \vee u = 9$
 $x^2 = -1 \vee x^2 = 9$
 $x = 3 \vee x = -3$

$$\begin{aligned}
 \text{c} \quad & x^4 + 16 = 10x^2 \\
 & x^4 - 10x^2 + 16 = 0 \\
 & \text{Stel } x^2 = u. \\
 & u^2 - 10u + 16 = 0 \\
 & (u - 2)(u - 8) = 0 \\
 & u = 2 \vee u = 8 \\
 & x^2 = 2 \vee x^2 = 8 \\
 & x = \sqrt{2} \vee x = -\sqrt{2} \vee x = \sqrt{8} \vee x = -\sqrt{8} \\
 & x = \sqrt{2} \vee x = -\sqrt{2} \vee x = 2\sqrt{2} \vee x = -2\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{d} \quad & x^3 + 25x = 10x^2 \\
 & x^3 - 10x^2 + 25x = 0 \\
 & x(x^2 - 10x + 25) = 0 \\
 & x(x - 5)^2 = 0 \\
 & x = 0 \vee x = 5
 \end{aligned}$$

$$\begin{aligned}
 34 \quad \text{a} \quad & 4x^4 + 153 = 53x^2 \\
 & 4x^4 - 53x^2 + 153 = 0 \\
 & \text{Stel } x^2 = u. \\
 & 4u^2 - 53u + 153 = 0 \\
 & D = (-53)^2 - 4 \cdot 4 \cdot 153 = 361 \\
 & u = \frac{53 \pm 19}{8} = 9 \vee u = \frac{53 - 19}{8} = 4\frac{1}{4} \\
 & x^2 = 9 \vee x^2 = 4\frac{1}{4} \\
 & x = 3 \vee x = -3 \vee x = \sqrt{4\frac{1}{4}} \vee x = -\sqrt{4\frac{1}{4}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b} \quad & 4x^4 + 21x^2 = 148 \\
 & 4x^4 + 21x^2 - 148 = 0 \\
 & \text{Stel } x^2 = u. \\
 & 4u^2 + 21u - 148 = 0 \\
 & D = 21^2 - 4 \cdot 4 \cdot (-148) = 2809 \\
 & u = \frac{-21 \pm 53}{8} = 4 \vee u = \frac{-21 - 53}{8} = -9\frac{1}{4} \\
 & x^2 = 4 \vee x^2 = -9\frac{1}{4} \\
 & x = 2 \vee x = -2
 \end{aligned}$$

$$\begin{aligned}
 \text{c} \quad & 4x^6 + 35 = 24x^3 \\
 & 4x^6 - 24x^3 + 35 = 0 \\
 & \text{Stel } x^3 = u. \\
 & 4u^2 - 24u + 35 = 0 \\
 & D = (-24)^2 - 4 \cdot 4 \cdot 35 = 16 \\
 & u = \frac{24 \pm 4}{8} = 3\frac{1}{2} \vee u = \frac{24 - 4}{8} = 2\frac{1}{2} \\
 & x^3 = 3\frac{1}{2} \vee x^3 = 2\frac{1}{2} \\
 & x = \sqrt[3]{3\frac{1}{2}} \vee x = \sqrt[3]{2\frac{1}{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{d} \quad & 64x^5 + 27x = 224x^3 \\
 & 64x^5 - 224x^3 + 27x = 0 \\
 & x(64x^4 - 224x^2 + 27) = 0 \\
 & x = 0 \vee 64x^4 - 224x^2 + 27 = 0 \\
 & \text{Stel } x^2 = u. \\
 & 64u^2 - 224u + 27 = 0 \\
 & D = (-224)^2 - 4 \cdot 64 \cdot 27 = 43\,264 \\
 & u = \frac{224 \pm 208}{128} = 3\frac{3}{8} \vee u = \frac{224 - 208}{128} = \frac{1}{8} \\
 & x^2 = 3\frac{3}{8} \vee x^2 = \frac{1}{8} \\
 & x = \sqrt{3\frac{3}{8}} \vee x = -\sqrt{3\frac{3}{8}} \vee x = \sqrt{\frac{1}{8}} \vee x = -\sqrt{\frac{1}{8}}
 \end{aligned}$$

35 Stel de lengte x en de breedte y .
Dan geldt $xy = 90$ en $x^2 + y^2 = 15^2$.

$$\text{Uit } xy = 90 \text{ volgt } y = \frac{90}{x}.$$

Substitutie van $y = \frac{90}{x}$ in $x^2 + y^2 = 15^2$ geeft

$$x^2 + \left(\frac{90}{x}\right)^2 = 15^2$$

$$x^2 + \frac{8100}{x^2} = 225$$

$$x^4 + 8100 = 225x^2$$

$$x^4 - 225x^2 + 8100 = 0$$

$$\text{Stel } x^2 = u.$$

$$u^2 - 225u + 8100 = 0$$

$$D = (-225)^2 - 4 \cdot 1 \cdot 8100 = 18\,225$$

$$u = \frac{225 \pm 135}{2} = 180 \vee u = \frac{225 - 135}{2} = 45$$

$$x^2 = 180 \vee x^2 = 45$$

$$x = \sqrt{180} \vee x = -\sqrt{180} \vee x = \sqrt{45} \vee x = -\sqrt{45}$$

$$x = 6\sqrt{5} \vee x = -6\sqrt{5} \text{ (kan niet)} \vee x = 3\sqrt{5} \vee x = -3\sqrt{5} \text{ (kan niet)}$$

$$x = 6\sqrt{5} \text{ geeft } y = \frac{90}{6\sqrt{5}} = \frac{90}{6\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{90\sqrt{5}}{30} = 3\sqrt{5}$$

$$x = 3\sqrt{5} \text{ geeft } y = \frac{90}{3\sqrt{5}} = \frac{90}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{90\sqrt{5}}{15} = 6\sqrt{5}$$

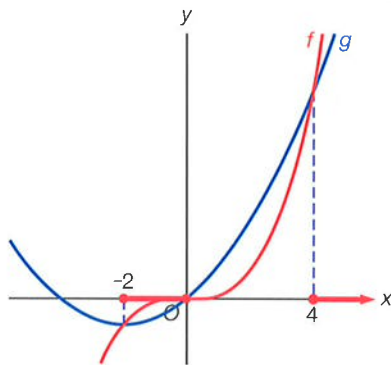
Dus de lengtes van de zijden van de rechthoek zijn $6\sqrt{5}$ en $3\sqrt{5}$.

Bladzijde 156

36 a $f(x) = g(x)$ geeft $x^4 = 2x^2 + 3$
 $x^4 - 2x^2 - 3 = 0$
 Stel $x^2 = u$.
 $u^2 - 2u - 3 = 0$
 $(u + 1)(u - 3) = 0$
 $u = -1 \vee u = 3$
 $x^2 = -1 \vee x^2 = 3$
 $x = \sqrt{3} \vee x = -\sqrt{3}$

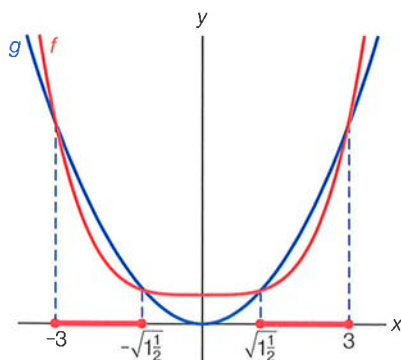
b $x^4 \leq 2x^2 + 3$ geeft $-\sqrt{3} \leq x \leq \sqrt{3}$

37 a Stel $f(x) = x^3$ en $g(x) = 2x^2 + 8x$.
 $f(x) = g(x)$ geeft $x^3 = 2x^2 + 8x$
 $x^3 - 2x^2 - 8x = 0$
 $x(x^2 - 2x - 8) = 0$
 $x(x + 2)(x - 4) = 0$
 $x = 0 \vee x = -2 \vee x = 4$



$x^3 \geq 2x^2 + 8x$ geeft $-2 \leq x \leq 0 \vee x \geq 4$

b Stel $f(x) = \frac{2}{3}x^4 + 9$ en $g(x) = 7x^2$.
 $f(x) = g(x)$ geeft $\frac{2}{3}x^4 + 9 = 7x^2$
 $2x^4 + 27 = 21x^2$
 $2x^4 - 21x^2 + 27 = 0$
 Stel $x^2 = u$.
 $2u^2 - 21u + 27 = 0$
 $D = (-21)^2 - 4 \cdot 2 \cdot 27 = 225$
 $u = \frac{21 \pm 15}{4} = 9 \vee u = \frac{21 - 15}{4} = 1\frac{1}{2}$
 $x^2 = 9 \vee x^2 = 1\frac{1}{2}$
 $x = 3 \vee x = -3 \vee x = \sqrt{1\frac{1}{2}} \vee x = -\sqrt{1\frac{1}{2}}$



$\frac{2}{3}x^4 + 9 \leq 7x^2$ geeft $-3 \leq x \leq -\sqrt{1\frac{1}{2}} \vee \sqrt{1\frac{1}{2}} \leq x \leq 3$

$$38 \quad x = -1 \text{ geeft } \begin{cases} 4 \cdot -1 - 5 = 9 \\ -4 - 5 = 9 \\ -9 = 9 \end{cases} \quad x = 3\frac{1}{2} \text{ geeft } \begin{cases} 4 \cdot 3\frac{1}{2} - 5 = 9 \\ 14 - 5 = 9 \\ 9 = 9 \end{cases}$$

en dat klopt.

en dat klopt.

Dus zowel $x = -1$ als $x = 3\frac{1}{2}$ is een oplossing.

$$39 \quad \text{a} \quad \begin{aligned} |2x - 1| &= 8 \\ 2x - 1 &= 8 \vee 2x - 1 = -8 \\ 2x &= 9 \vee 2x = -7 \\ x &= 4\frac{1}{2} \vee x = -3\frac{1}{2} \end{aligned}$$

$$\text{b} \quad \begin{aligned} |x^2 - 3| &= 1 \\ x^2 - 3 &= 1 \vee x^2 - 3 = -1 \\ x^2 &= 4 \vee x^2 = 2 \\ x &= 2 \vee x = -2 \vee x = \sqrt{2} \vee x = -\sqrt{2} \end{aligned}$$

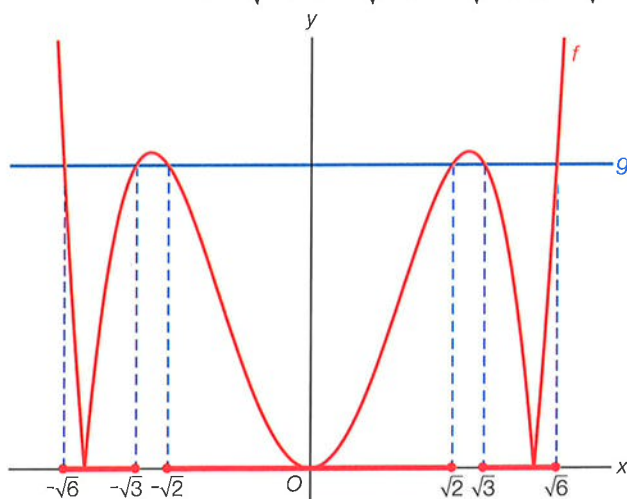
$$\begin{aligned} \text{c} \quad &|2x^2 - 5| = 11 \\ &2x^2 - 5 = 11 \vee 2x^2 - 5 = -11 \\ &2x^2 = 16 \vee 2x^2 = -6 \\ &x^2 = 8 \vee x^2 = -3 \\ &x = \sqrt{8} = 2\sqrt{2} \vee x = -\sqrt{8} = -2\sqrt{2} \\ \text{d} \quad &|5 - x^2| = 11 \\ &5 - x^2 = 11 \vee 5 - x^2 = -11 \\ &-x^2 = 6 \vee -x^2 = -16 \\ &x^2 = -6 \vee x^2 = 16 \\ &x = 4 \vee x = -4 \end{aligned}$$

$$40 \quad \text{a} \quad \begin{aligned} |2x^4 - 5| &= 15 \\ 2x^4 - 5 &= 15 \vee 2x^4 - 5 = -15 \\ 2x^4 &= 20 \vee 2x^4 = -10 \\ x^4 &= 10 \vee x^4 = -5 \\ x &= \sqrt[4]{10} \vee x = -\sqrt[4]{10} \end{aligned}$$

$$\text{b} \quad \begin{aligned} |2x^3 - 5| &= 15 \\ 2x^3 - 5 &= 15 \vee 2x^3 - 5 = -15 \\ 2x^3 &= 20 \vee 2x^3 = -10 \\ x^3 &= 10 \vee x^3 = -5 \\ x &= \sqrt[3]{10} \vee x = \sqrt[3]{-5} \end{aligned}$$

$$\text{c} \quad \text{Stel } f(x) = |x^4 - 5x^2| \text{ en } g(x) = 6.$$

$$\begin{aligned} f(x) = g(x) \text{ geeft } |x^4 - 5x^2| &= 6 \\ x^4 - 5x^2 &= 6 \vee x^4 - 5x^2 = -6 \\ x^4 - 5x^2 - 6 &= 0 \vee x^4 - 5x^2 + 6 = 0 \\ \text{Stel } x^2 &= u. \\ u^2 - 5u - 6 &= 0 \vee u^2 - 5u + 6 &= 0 \\ (u + 1)(u - 6) &= 0 \vee (u - 2)(u - 3) &= 0 \\ u = -1 \vee u &= 6 \vee u = 2 \vee u &= 3 \\ x^2 = -1 \vee x^2 &= 6 \vee x^2 = 2 \vee x^2 &= 3 \\ x = \sqrt{6} \vee x &= -\sqrt{6} \vee x = \sqrt{2} \vee x &= -\sqrt{2} \vee x = \sqrt{3} \vee x = -\sqrt{3} \end{aligned}$$



$$|x^4 - 5x^2| \leq 6 \text{ geeft } -\sqrt{6} \leq x \leq -\sqrt{3} \vee -\sqrt{2} \leq x \leq \sqrt{2} \vee \sqrt{3} \leq x \leq \sqrt{6}$$

- d Stel $f(x) = |x^6 - 10x^3|$ en $g(x) = 24$.

$$f(x) = g(x) \text{ geeft } |x^6 - 10x^3| = 24$$

$$x^6 - 10x^3 = 24 \vee x^6 - 10x^3 = -24$$

$$x^6 - 10x^3 - 24 = 0 \vee x^6 - 10x^3 + 24 = 0$$

$$\text{Stel } x^3 = u.$$

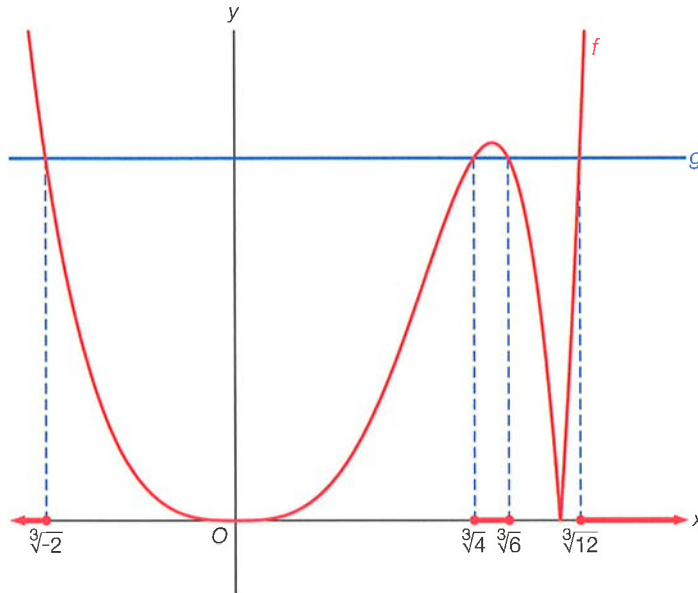
$$u^2 - 10u - 24 = 0 \vee u^2 - 10u + 24 = 0$$

$$(u + 2)(u - 12) = 0 \vee (u - 4)(u - 6) = 0$$

$$u = -2 \vee u = 12 \vee u = 4 \vee u = 6$$

$$x^3 = -2 \vee x^3 = 12 \vee x^3 = 4 \vee x^3 = 6$$

$$x = \sqrt[3]{-2} \vee x = \sqrt[3]{12} \vee x = \sqrt[3]{4} \vee x = \sqrt[3]{6}$$



$$|x^6 - 10x^3| \geq 24 \text{ geeft } x \leq \sqrt[3]{-2} \vee \sqrt[3]{4} \leq x \leq \sqrt[3]{6} \vee x \geq \sqrt[3]{12}$$

4.3 Regels voor het oplossen van vergelijkingen

Bladzijde 159

41 a $5x(x^2 - 4) = 15(x^2 - 4)$

$$5x^3 - 20x = 15x^2 - 60$$

$$5x^3 - 15x^2 - 20x + 60 = 0$$

Dit is geen goede aanpak omdat je geen methode kent om deze derdegraadsvergelijking algebraïsch op te lossen.

b $x = 2$ invullen in $5x(x^2 - 4) = 15(x^2 - 4)$ geeft $5 \cdot 2(2^2 - 4) = 15(2^2 - 4)$

$$10(4 - 4) = 15(4 - 4)$$

$$10 \cdot 0 = 15 \cdot 0$$

$$0 = 0$$

en dat klopt.

Dus ik ben het daarmee eens.

c $5x = 15$ geeft de oplossing $x = 3$.

Maar ook $x = 2$ en $x = -2$ zijn oplossingen.

Door het linker- en rechterlid door $x^2 - 4$ te delen, zijn deze oplossingen echter verloren gegaan.

Bladzijde 161

42 a $(4x - 1)^2 = (3x - 2)^2$

$$4x - 1 = 3x - 2 \vee 4x - 1 = -3x + 2$$

$$x = -1 \vee 7x = 3$$

$$x = -1 \vee x = \frac{3}{7}$$

b $(3x^2 - 5)^2 = 4x^2$

$$3x^2 - 5 = 2x$$

$$\vee 3x^2 - 5 = -2x$$

$$3x^2 - 2x - 5 = 0$$

$$\vee 3x^2 + 2x - 5 = 0$$

$$D = (-2)^2 - 4 \cdot 3 \cdot -5 = 64$$

$$\vee D = 2^2 - 4 \cdot 3 \cdot -5 = 64$$

$$x = \frac{2+8}{6} = 1\frac{2}{3} \vee x = \frac{2-8}{6} = -1 \vee x = \frac{-2+8}{6} = 1 \vee x = \frac{-2-8}{6} = -1\frac{2}{3}$$

c $(x^2 - 4x)(x^2 - 8) = 0$

$$x^2 - 4x = 0 \vee x^2 - 8 = 0$$

$$x(x - 4) = 0 \vee x^2 = 8$$

$$x = 0 \vee x = 4 \vee x = \sqrt{8} \vee x = -\sqrt{8}$$

$$x = 0 \vee x = 4 \vee x = 2\sqrt{2} \vee x = -2\sqrt{2}$$

d $x^3(x - 3) = 8(x - 3)$

$$x - 3 = 0 \vee x^3 = 8$$

$$x = 3 \vee x = 2$$

e $(3x - 2)^4 = 3x - 2$

$$(3x - 2)(3x - 2)^3 = 3x - 2$$

$$3x - 2 = 0 \vee (3x - 2)^3 = 1$$

$$3x = 2 \vee 3x - 2 = 1$$

$$x = \frac{2}{3} \vee 3x = 3$$

$$x = \frac{2}{3} \vee x = 1$$

f $2x(x^2 - 4) = 6(x - 2)$

$$2x(x + 2)(x - 2) = 6(x - 2)$$

$$x - 2 = 0 \vee 2x(x + 2) = 6$$

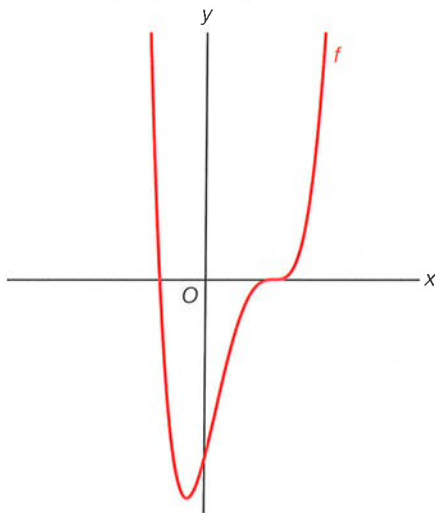
$$x = 2 \vee 2x^2 + 4x - 6 = 0$$

$$x = 2 \vee x^2 + 2x - 3 = 0$$

$$x = 2 \vee (x - 1)(x + 3) = 0$$

$$x = 2 \vee x = 1 \vee x = -3$$

43 a Voer in $y_1 = (3x + 4)(x - 2)^3$.



De optie minimum geeft $x \approx -0,5$ en $y \approx -39,1$, dus $B_f = [-39,1; \rightarrow)$.

b $(3x + 4)(x - 2)^3 = 0$

$$3x + 4 = 0 \vee (x - 2)^3 = 0$$

$$3x = -4 \vee x - 2 = 0$$

$$x = -1\frac{1}{3} \vee x = 2$$

Dus de nulpunten zijn $-1\frac{1}{3}$ en 2.

c $(3x + 4)(x - 2)^3 = 3x + 4$

$$3x + 4 = 0 \vee (x - 2)^3 = 1$$

$$3x = -4 \vee x - 2 = 1$$

$$x = -1\frac{1}{3} \vee x = 3$$

$x = -1\frac{1}{3}$ geeft $y = 0$ (zie a), dus $A(-1\frac{1}{3}, 0)$.

$x = 3$ geeft $y = 3 \cdot 3 + 4 = 13$, dus $B(3, 13)$.

- d** $(3x+4)(x-2)^3 = (3x+4)(x-2)$
 $(3x+4)(x-2)(x-2)^2 = (3x+4)(x-2)$
 $(3x+4)(x-2) = 0 \vee (x-2)^2 = 1$
 $3x = -4 \vee x = 2 \vee x - 2 = 1 \vee x - 2 = -1$
 $x = -1\frac{1}{3} \vee x = 2 \vee x = 3 \vee x = 1$
 $x = -1\frac{1}{3}$ geeft $y = 0$ (zie a)
 $x = 1$ geeft $y = (3 \cdot 1 + 4)(1 - 2) = -7$
 $x = 2$ geeft $y = 0$ (zie a)
 $x = 3$ geeft $y = 13$ (zie c)
 Dus de snijpunten zijn $(-1\frac{1}{3}, 0)$, $(1, -7)$, $(2, 0)$ en $(3, 13)$.

- 44** $x = 1$ geeft $(4+p)^3 = 4+p$
 $(4+p)(4+p)^2 = 4+p$
 $4+p = 0 \vee (4+p)^2 = 1$
 $p = -4 \vee 4+p = 1 \vee 4+p = -1$
 $p = -4 \vee p = -3 \vee p = -5$
 $(4x+p)^3 = 4x+p$
 $(4x+p)(4x+p)^2 = 4x+p$
 $4x+p = 0 \vee (4x+p)^2 = 1$
 $4x = -p \vee 4x+p = 1 \vee 4x+p = -1$
 $x = -\frac{1}{4}p \vee 4x = 1-p \vee 4x = -1-p$
 $x = -\frac{1}{4}p \vee x = \frac{1-p}{4} \vee x = \frac{-1-p}{4}$
 $p = -4$ geeft $x = 1 \vee x = 1\frac{1}{4} \vee x = \frac{3}{4}$
 $p = -3$ geeft $x = \frac{3}{4} \vee x = 1 \vee x = \frac{1}{2}$
 $p = -5$ geeft $x = 1\frac{1}{4} \vee x = 1\frac{1}{2} \vee x = 1$
 Dus de andere oplossingen zijn $x = \frac{1}{2}$, $x = \frac{3}{4}$, $x = 1\frac{1}{4}$ en $x = 1\frac{1}{2}$.

- 45 a** De wortel van 9 is 3, dus $2x - 5 = 9$
 $2x = 14$
 $x = 7$
b Een wortel kan niet negatief zijn, dus $\sqrt{2x-5}$ is voor geen enkele x gelijk aan -3 .
 Dus de vergelijking $\sqrt{2x-5} = -3$ heeft geen oplossing.

Bladzijde 163

- 46 a** $x = \sqrt{5x+14}$
 kwadrateren geeft
 $x^2 = 5x + 14$
 $x^2 - 5x - 14 = 0$
 $(x+2)(x-7) = 0$
 $x = -2 \vee x = 7$
 $x = -2$ geeft $-2 = \sqrt{-10+14}$ vold. niet
 $x = 7$ geeft $7 = \sqrt{35+14}$ vold.
b $3x = \sqrt{8x+20}$
 kwadrateren geeft
 $9x^2 = 8x + 20$
 $9x^2 - 8x - 20 = 0$
 $D = (-8)^2 - 4 \cdot 9 \cdot -20 = 784$
 $x = \frac{8+28}{18} = 2 \vee x = \frac{8-28}{18} = -1\frac{1}{9}$
 $x = 2$ geeft $6 = \sqrt{16+20}$ vold.
 $x = -1\frac{1}{9}$ geeft $-3\frac{1}{3} = \sqrt{-8\frac{8}{9}+20}$ vold. niet
c $5\sqrt{x} = x$
 kwadrateren geeft
 $25x = x^2$
 $x^2 - 25x = 0$
 $x(x-25) = 0$
 $x = 0 \vee x = 25$
 $x = 0$ geeft $5\sqrt{0} = 0$ vold.
 $x = 25$ geeft $5\sqrt{25} = 25$ vold.
d $3x = \sqrt{18x+72}$
 kwadrateren geeft
 $9x^2 = 18x + 72$
 $9x^2 - 18x - 72 = 0$
 $x^2 - 2x - 8 = 0$
 $(x+2)(x-4) = 0$
 $x = -2 \vee x = 4$
 $x = -2$ geeft $-6 = \sqrt{-36+72}$ vold. niet
 $x = 4$ geeft $12 = \sqrt{72+72}$ vold.

47 a $4 - 3\sqrt{x} = 2$
 $-3\sqrt{x} = -2$
 $\sqrt{x} = \frac{2}{3}$
 $x = \frac{4}{9}$

b $5\sqrt{x} - 2x = 0$
 $5\sqrt{x} = 2x$
kwadrateren geeft
 $25x = 4x^2$
 $4x^2 - 25x = 0$
 $x(4x - 25) = 0$
 $x = 0 \vee 4x = 25$
 $x = 0 \vee x = 6\frac{1}{4}$
 $x = 0$ geeft $5\sqrt{0} - 0 = 0$ vold.
 $x = 6\frac{1}{4}$ geeft $5\sqrt{6\frac{1}{4}} - 12\frac{1}{2} = 0$ vold.

48 a $2x + \sqrt{x} = 10$
 $\sqrt{x} = 10 - 2x$
kwadrateren geeft
 $x = (10 - 2x)^2$
 $x = 100 - 40x + 4x^2$
 $4x^2 - 41x + 100 = 0$
 $D = (-41)^2 - 4 \cdot 4 \cdot 100 = 81$
 $x = \frac{41 \pm 9}{8} = 6\frac{1}{4} \vee x = \frac{41 - 9}{8} = 4$
 $x = 6\frac{1}{4}$ geeft $12\frac{1}{2} + \sqrt{6\frac{1}{4}} = 10$ vold. niet
 $x = 4$ geeft $8 + \sqrt{4} = 10$ vold.

b $\sqrt{x+12} = x$
kwadrateren geeft
 $x + 12 = x^2$
 $x^2 - x - 12 = 0$
 $(x+3)(x-4) = 0$
 $x = -3 \vee x = 4$
 $x = -3$ geeft $\sqrt{-3+12} = -3$ vold. niet
 $x = 4$ geeft $\sqrt{4+12} = 4$ vold.

49 a $\sqrt{x} = u$, dus $(\sqrt{x})^2 = u^2$ oftewel $x = u^2$.
Dit geeft $u^2 - u = 12$

$$u^2 - u - 12 = 0$$

$$(u+3)(u-4) = 0$$

$$u = -3 \vee u = 4$$

$$\sqrt{x} = -3 \vee \sqrt{x} = 4$$

$$x = 16$$

b $x\sqrt{x} = u$, dus $(x\sqrt{x})^2 = u^2$ oftewel $x^3 = u^2$.
Dit geeft $u^2 - 9u + 8 = 0$

$$(u-1)(u-8) = 0$$

$$u = 1 \vee u = 8$$

$$x\sqrt{x} = 1 \vee x\sqrt{x} = 8$$

$$x^3 = 1 \vee x^3 = 64$$

$$x = 1 \vee x = 4$$

$$x = 1 \text{ geeft } 1^3 - 9 \cdot 1 \cdot \sqrt{1} + 8 = 0 \text{ vold.}$$

$$x = 4 \text{ geeft } 4^3 - 9 \cdot 4 \cdot \sqrt{4} + 8 = 0 \text{ vold.}$$

c $2x - 5\sqrt{x} = 3$
 $2x - 3 = 5\sqrt{x}$
kwadrateren geeft
 $(2x - 3)^2 = 25x$
 $4x^2 - 12x + 9 = 25x$
 $4x^2 - 37x + 9 = 0$
 $D = (-37)^2 - 4 \cdot 4 \cdot 9 = 1225$
 $x = \frac{37 \pm 35}{8} = 9 \vee x = \frac{37 - 35}{8} = \frac{1}{4}$
 $x = 9$ geeft $18 - 5\sqrt{9} = 3$ vold.
 $x = \frac{1}{4}$ geeft $\frac{1}{2} - 5\sqrt{\frac{1}{4}} = 3$ vold. niet

d $5 - 2\sqrt{x} = 3$
 $-2\sqrt{x} = -2$
 $\sqrt{x} = 1$
 $x = 1$

c $2x + \sqrt{x} = 6$
 $\sqrt{x} = 6 - 2x$
kwadrateren geeft
 $x = (6 - 2x)^2$
 $x = 36 - 24x + 4x^2$
 $4x^2 - 25x + 36 = 0$
 $D = (-25)^2 - 4 \cdot 4 \cdot 36 = 49$
 $x = \frac{25 \pm 7}{8} = 4 \vee x = \frac{25 - 7}{8} = 2\frac{1}{4}$
 $x = 4$ geeft $8 + \sqrt{4} = 6$ vold. niet
 $x = 2\frac{1}{4}$ geeft $4\frac{1}{2} + \sqrt{2\frac{1}{4}} = 6$ vold.

d $10 - x\sqrt{x} = 2$
 $-x\sqrt{x} = -8$
kwadrateren geeft
 $x^3 = 64$
 $x = 4$
 $x = 4$ geeft $10 - 4\sqrt{4} = 2$ vold.

- c Stel $x^2\sqrt{x} = u$, dan is $(x^2\sqrt{x})^2 = u^2$ oftewel $x^5 = u^2$.

Dit geeft $u^2 + 10 = 7u$

$$u^2 - 7u + 10 = 0$$

$$(u - 2)(u - 5) = 0$$

$$u = 2 \vee u = 5$$

$$x^2\sqrt{x} = 2 \vee x^2\sqrt{x} = 5$$

$$x^5 = 4 \vee x^5 = 25$$

$$x = \sqrt[5]{4} \vee x = \sqrt[5]{25}$$

$$x = \sqrt[5]{4} \text{ geeft } (\sqrt[5]{4})^5 + 10 = 7 \cdot (\sqrt[5]{4})^2 \cdot \sqrt[5]{4} \text{ vold.}$$

$$x = \sqrt[5]{25} \text{ geeft } (\sqrt[5]{25})^5 + 10 = 7 \cdot (\sqrt[5]{25})^2 \cdot \sqrt[5]{25} \text{ vold.}$$

Bladzijde 164

- 50 a Bij een verhoudingstabel zijn de kruisproducten gelijk.

$$\text{Dus } \begin{array}{c|c} x & x+4 \\ \hline 2 & x \end{array} \text{ geeft } x \cdot x = 2 \cdot (x+4)$$

$$x^2 = 2x + 8$$

$$x^2 - 2x - 8 = 0$$

- b Oplossen van $x^2 - 2x - 8 = 0$ geeft $(x+2)(x-4) = 0$

$$x = -2 \vee x = 4$$

Bladzijde 166

51 a $\frac{x-3}{x+1} = 1\frac{1}{2}$

$$(x-3) = 1\frac{1}{2}(x+1)$$

$$2(x-3) = 3(x+1)$$

$$2x - 6 = 3x + 3$$

$$-x = 9$$

$$x = -9$$

vold.

b $\frac{x-1}{x} + 1 = 3$

$$\frac{x-1}{x} = 2$$

$$x-1 = 2x$$

$$-x = 1$$

$$x = -1$$

vold.

c $\frac{3x+4}{x-1} = \frac{x+18}{x}$

$$x(3x+4) = (x-1)(x+18)$$

$$3x^2 + 4x = x^2 + 18x - x - 18$$

$$2x^2 - 13x + 18 = 0$$

$$D = (-13)^2 - 4 \cdot 2 \cdot 18 = 25$$

$$x = \frac{13+5}{4} = 4\frac{1}{2} \vee x = \frac{13-5}{4} = 2$$

vold.

vold.

d $\frac{2x-5}{4-x} = \frac{x+2}{3x-4}$

$$(2x-5)(3x-4) = (4-x)(x+2)$$

$$6x^2 - 8x - 15x + 20 = 4x + 8 - x^2 - 2x$$

$$7x^2 - 25x + 12 = 0$$

$$D = (-25)^2 - 4 \cdot 7 \cdot 12 = 289$$

$$x = \frac{25+17}{14} = 3 \vee x = \frac{25-17}{14} = \frac{4}{7}$$

vold.

vold.

52 a $\frac{5x^2-15}{x^2+5} = 0$

$$5x^2 - 15 = 0$$

$$5x^2 = 15$$

$$x^2 = 3$$

$$x = \sqrt{3} \vee x = -\sqrt{3}$$

vold. vold.

b $\frac{x^2-3}{x^2+1} = \frac{x-1}{x^2+1}$

$$x^2 - 3 = x - 1$$

$$x^2 - x - 2 = 0$$

$$(x+1)(x-2) = 0$$

$$x = -1 \vee x = 2$$

vold. vold.

c $\frac{x^2-4}{2x+5} = \frac{x^2-4}{x+4}$

$$x^2 - 4 = 0 \vee 2x + 5 = x + 4$$

$$x^2 = 4 \vee x = -1$$

$$x = 2 \vee x = -2 \vee x = -1$$

vold. vold. vold.

d $\frac{x^2+1}{x+1} = \frac{x+3}{x+1}$

$$x^2 + 1 = x + 3$$

$$x^2 - x - 2 = 0$$

$$(x+1)(x-2) = 0$$

$$x = -1 \vee x = 2$$

vold. niet vold.

53 a $\frac{3x^2 - 10}{x^2 + 1} = 2$
 $3x^2 - 10 = 2(x^2 + 1)$
 $3x^2 - 10 = 2x^2 + 2$
 $x^2 = 12$
 $x = \sqrt{12} = 2\sqrt{3} \vee x = -2\sqrt{3}$
 vold. vold.

b $\frac{x^3 - 8}{x^2 + 2} = \frac{x^3 - 8}{x + 8}$
 $x^3 - 8 = 0 \vee x^2 + 2 = x + 8$
 $x^3 = 8 \vee x^2 - x - 6 = 0$
 $x = 2 \vee (x + 2)(x - 3) = 0$
 $x = 2 \vee x = -2 \vee x = 3$
 vold. vold. vold.

c $\frac{3x^2 - 10}{(x^2 + 1)^2} = \frac{2}{25}$
 $2(x^2 + 1)^2 = 25(3x^2 - 10)$
 $2(x^4 + 2x^2 + 1) = 75x^2 - 250$
 $2x^4 + 4x^2 + 2 = 75x^2 - 250$
 $2x^4 - 71x^2 + 252 = 0$
 Stel $x^2 = u$.
 $2u^2 - 71u + 252 = 0$
 $D = (-71)^2 - 4 \cdot 2 \cdot 252 = 3025$
 $u = \frac{71 \pm 55}{4} = 31\frac{1}{2} \vee u = \frac{71 - 55}{4} = 4$
 $x^2 = 31\frac{1}{2} \vee x^2 = 4$
 $x = \sqrt{31\frac{1}{2}} \vee x = -\sqrt{31\frac{1}{2}} \vee x = 2 \vee x = -2$
 vold. vold. vold. vold.

d $\frac{6x^2 - 12}{(x^2 - 1)^2} = 1\frac{1}{3}$
 $\frac{6x^2 - 12}{(x^2 - 1)^2} = \frac{4}{3}$
 $4(x^2 - 1)^2 = 3(6x^2 - 12)$
 $4(x^4 - 2x^2 + 1) = 18x^2 - 36$
 $4x^4 - 8x^2 + 4 = 18x^2 - 36$
 $4x^4 - 26x^2 + 40 = 0$
 $2x^4 - 13x^2 + 20 = 0$
 Stel $x^2 = u$.
 $2u^2 - 13u + 20 = 0$
 $D = (-13)^2 - 4 \cdot 2 \cdot 20 = 9$
 $u = \frac{13 \pm 3}{4} = 4 \vee u = \frac{13 - 3}{4} = 2\frac{1}{2}$
 $x^2 = 4 \vee x^2 = 2\frac{1}{2}$
 $x = 2 \vee x = -2 \vee x = \sqrt{2\frac{1}{2}} \vee x = -\sqrt{2\frac{1}{2}}$
 vold. vold. vold. vold.

54 $f(x) = \frac{-10x}{x^2 + 2}$ geeft $f'(x) = \frac{(x^2 + 2) \cdot -10 + 10x \cdot 2x}{(x^2 + 2)^2} = \frac{-10x^2 - 20 + 20x^2}{(x^2 + 2)^2} = \frac{10x^2 - 20}{(x^2 + 2)^2}$
 $f'(x) = \frac{5}{9}$ geeft $\frac{10x^2 - 20}{(x^2 + 2)^2} = \frac{5}{9}$
 $5(x^2 + 2)^2 = 9(10x^2 - 20)$
 $5(x^4 + 4x^2 + 4) = 90x^2 - 180$
 $5x^4 + 20x^2 + 20 = 90x^2 - 180$
 $5x^4 - 70x^2 + 200 = 0$
 $x^4 - 14x^2 + 40 = 0$
 Stel $x^2 = u$.
 $u^2 - 14u + 40 = 0$
 $(u - 4)(u - 10) = 0$
 $u = 4 \vee u = 10$
 $x^2 = 4 \vee x^2 = 10$
 $x = 2 \vee x = -2 \vee x = \sqrt{10} \vee x = -\sqrt{10}$
 vold. vold. vold. vold.

- 55** Stel dat Wouter na 1 km te hebben gelopen zich x km van de sportclub bevindt.
Stel dat hij loopt met een snelheid van l km/uur, dan fietst hij met een snelheid van $7l$ km/uur.

Als hij doorloopt, legt hij $1 + x$ km lopend af in $\frac{1+x}{l}$ uur.

Als hij terugloopt en de fiets pakt, legt hij 2 km lopend en $1 + x$ km fietsend af in $\frac{2}{l} + \frac{1+x}{7l}$ uur.

$$\frac{1+x}{l} = \frac{2}{l} + \frac{1+x}{7l}$$

$$7l \cdot \frac{1+x}{l} = 7l \cdot \frac{2}{l} + 7l \cdot \frac{1+x}{7l}$$

$$7(1+x) = 7 \cdot 2 + 1+x$$

$$7 + 7x = 14 + 1 + x$$

$$6x = 8$$

$$x = 1\frac{1}{3}$$

Dus hij is op dat moment $1\frac{1}{3}$ km van zijn sportclub verwijderd.

4.4 Herleidingen en inverse functies

Bladzijde 168

- 56** $x^2 + 3x + 2 = (x+1)(x+2)$ en $x^2 + 2x + 1 = (x+1)^2$, dus $\frac{x^2 + 3x + 2}{x^2 + 2x + 2} = \frac{(x+1)(x+2)}{(x+1)^2}$.

Teller en noemer delen door $x+1$ geeft $\frac{x+2}{x+1}$.

$$\text{Dus } \frac{x^2 + 3x + 2}{x^2 + 2x + 2} = \frac{(x+1)(x+2)}{(x+1)^2} = \frac{x+2}{x+1}.$$

Bladzijde 169

- 57 a** $\frac{x^3 - 2x^2}{x^2 + 2x} = \frac{x^2(x-2)}{x(x+2)} = \frac{x(x-2)}{x+2}$ mits $x \neq 0$

b $\frac{x^2 - 4x}{x^2 - 16} = \frac{x(x-4)}{(x+4)(x-4)} = \frac{x}{x+4}$ mits $x \neq 4$

c $\frac{x^4 - 3x^2 - 4}{x^4 - 1} = \frac{(x^2+1)(x^2-4)}{(x^2+1)(x^2-1)} = \frac{x^2-4}{x^2-1}$

- 58 a** $\frac{2x^5 - 32x}{x^2 - 4} = \frac{2x(x^4 - 16)}{x^2 - 4} = \frac{2x(x^2+4)(x^2-4)}{x^2 - 4} = 2x(x^2+4)$ mits $x \neq 2 \wedge x \neq -2$

b $\frac{x^4 + 4x^2 + 4}{x^4 - 4} = \frac{(x^2+2)(x^2+2)}{(x^2+2)(x^2-2)} = \frac{x^2+2}{x^2-2}$

c $\frac{x^4 - 9x^2}{x^2 - 3x} = \frac{x^2(x^2-9)}{x(x-3)} = \frac{x^2(x+3)(x-3)}{x(x-3)} = x(x+3)$ mits $x \neq 0 \wedge x \neq 3$

- 59 a** $x_P = p$ geeft $y_P = p^2$, dus $P(p, p^2)$.
 $x_Q = q$ geeft $y_Q = q^2$, dus $Q(q, q^2)$.

$$\text{Stel } k: y = ax + b \text{ met } a = \frac{\Delta y}{\Delta x} = \frac{q^2 - p^2}{q - p} = \frac{(q+p)(q-p)}{q-p} = q + p.$$

$$\left. \begin{array}{l} y = (p+q)x + b \\ P(p, p^2) \end{array} \right\} \begin{array}{l} (p+q) \cdot p + b = p^2 \\ p^2 + pq + b = p^2 \\ b = -pq \end{array}$$

$$\text{Dus } k: y = (p+q)x - pq.$$

- b** $f(a) = a^4$, dus $A(a, a^4)$.
 $f(b) = b^4$, dus $B(b, b^4)$.

Stel $l: y = mx + n$ met

$$m = \frac{\Delta y}{\Delta x} = \frac{b^4 - a^4}{b - a} = \frac{(b^2 + a^2)(b^2 - a^2)}{b - a} = \frac{(b^2 + a^2)(b + a)(b - a)}{b - a} \\ = (b^2 + a^2)(b + a) = b^3 + ab^2 + a^2b + a^3.$$

$$y = (a^3 + a^2b + ab^2 + b^3)x + n \quad \left\{ \begin{array}{l} (a^3 + a^2b + ab^2 + b^3) \cdot a + n = a^4 \\ a^4 + a^3b + a^2b^2 + ab^3 + n = a^4 \\ n = -a^3b - a^2b^2 - ab^3 \\ n = -ab(a^2 + ab + b^2) \end{array} \right.$$

Dus $l: y = (a^3 + a^2b + ab^2 + b^3)x - ab(a^2 + ab + b^2)$.

60 a $2x - \frac{1}{x} = \frac{2x}{1} - \frac{1}{x} = \frac{2x^2}{x} - \frac{1}{x} = \frac{2x^2 - 1}{x}$

b $(x+1) \cdot \frac{x+2}{x+3} = \frac{(x+1)(x+2)}{x+3} = \frac{x^2 + 2x + x + 2}{x+3} = \frac{x^2 + 3x + 2}{x+3}$

c $\frac{x}{\left(\frac{2}{x}\right)} = x \cdot \frac{x}{2} = \frac{x^2}{2} = \frac{1}{2}x^2$ mits $x \neq 0$

Dus de herleiding is niet juist voor $x = 0$.

d $y = \frac{2x + \frac{1}{3}}{x+1} = \frac{2x + \frac{1}{3}}{x+1} \cdot \frac{3}{3} = \frac{3(2x + \frac{1}{3})}{3(x+1)} = \frac{6x+1}{3x+3}$

Bladzijde 171

61 a $y = \frac{20}{x} - \frac{5}{2x} = \frac{40}{2x} - \frac{5}{2x} = \frac{35}{2x}$

b $y = \frac{10}{x-1} - x^2 = \frac{10}{x-1} - \frac{x^2(x-1)}{x-1} = \frac{10 - x^2(x-1)}{x-1} = \frac{10 - x^3 + x^2}{x-1}$

c $y = \frac{2x^2}{\left(\frac{x+1}{x-1}\right)} = 2x^2 \cdot \frac{x-1}{x+1} = \frac{2x^2(x-1)}{x+1} = \frac{2x^3 - 2x^2}{x+1}$ mits $x \neq 1$

d $y = \frac{x}{x-1} \left(x + \frac{1}{x-1} \right) = \frac{x}{x-1} \left(\frac{x(x-1)}{x-1} + \frac{1}{x-1} \right) = \frac{x}{x-1} \cdot \frac{x^2 - x + 1}{x-1} = \frac{x^3 - x^2 + x}{(x-1)^2}$

e $y = \frac{5}{x-2} \cdot \frac{6}{x+2} = \frac{30}{(x-2)(x+2)}$

f $y = \frac{\left(\frac{x+1}{2x}\right)}{x-1} = \frac{x+1}{2x(x-1)}$

62 a $y = \frac{x + \frac{3}{x+1}}{x} = \frac{\left(x + \frac{3}{x+1}\right) \cdot (x+1)}{x \cdot (x+1)} = \frac{x(x+1) + 3}{x(x+1)} = \frac{x^2 + x + 3}{x(x+1)}$

b $y = \frac{10 + \frac{5}{x-1}}{6 - \frac{3}{x-1}} = \frac{\left(10 + \frac{5}{x-1}\right) \cdot (x-1)}{\left(6 - \frac{3}{x-1}\right) \cdot (x-1)} = \frac{10(x-1) + 5}{6(x-1) - 3} = \frac{10x - 5}{6x - 9}$ mits $x \neq 1$

c $Q = \frac{10}{p + \frac{5}{2p}} = \frac{10 \cdot 2p}{\left(p + \frac{5}{2p}\right) \cdot 2p} = \frac{20p}{2p^2 + 5}$ mits $p \neq 0$

63 a $y = \frac{5x^2 + 1000}{x} = 5x + \frac{1000}{x}$

b $K = \frac{6t^2 + 12t + 1500}{3t} = 2t + 4 + \frac{500}{t}$

$$\text{c } F = \frac{5a^2 + 8a}{2a^2} = 2\frac{1}{2} + \frac{4}{a}$$

$$\text{d } N = \frac{6p^2 - 3p - 1}{2p} = 3p - 1\frac{1}{2} - \frac{1}{2p}$$

64 a $f(x) = g(x)$ geeft $\frac{4x}{x+1} = \frac{4}{x-1}$

$$4x(x-1) = 4(x+1)$$

$$4x^2 - 4x = 4x + 4$$

$$4x^2 - 8x - 4 = 0$$

$$x^2 - 2x - 1 = 0$$

$$D = (-2)^2 - 4 \cdot 1 \cdot -1 = 8, \text{ dus } \sqrt{D} = 2\sqrt{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2} \vee x = 1 - \sqrt{2}$$

vold. vold.

b $f(x) \cdot g(x) = 6$ geeft $\frac{4x}{x+1} \cdot \frac{4}{x-1} = 6$

$$\frac{16x}{x^2 - 1} = 6$$

$$16x = 6(x^2 - 1)$$

$$16x = 6x^2 - 6$$

$$6x^2 - 16x - 6 = 0$$

$$3x^2 - 8x - 3 = 0$$

$$D = (-8)^2 - 4 \cdot 3 \cdot -3 = 100$$

$$x = \frac{8 \pm 10}{6} = 3 \vee x = \frac{8 - 10}{6} = -\frac{1}{3}$$

vold. vold.

c $f(x) - g(x) = 7$ geeft $\frac{4x}{x+1} - \frac{4}{x-1} = 7$

$$\frac{4x(x-1)}{(x+1)(x-1)} - \frac{4(x+1)}{(x+1)(x-1)} = 7$$

$$\frac{4x^2 - 4x - 4x - 4}{(x+1)(x-1)} = 7$$

$$\frac{4x^2 - 8x - 4}{x^2 - 1} = 7$$

$$4x^2 - 8x - 4 = 7(x^2 - 1)$$

$$4x^2 - 8x - 4 = 7x^2 - 7$$

$$-3x^2 - 8x + 3 = 0$$

$$D = (-8)^2 - 4 \cdot -3 \cdot 3 = 100$$

$$x = \frac{8 \pm 10}{-6} = -3 \vee x = \frac{8 - 10}{-6} = \frac{1}{3}$$

vold. vold.

d $f(x) = \frac{4x}{x+1}$ geeft $f'(x) = \frac{(x+1) \cdot 4 - 4x \cdot 1}{(x+1)^2} = \frac{4x + 4 - 4x}{(x+1)^2} = \frac{4}{(x+1)^2}$

$g(x) = \frac{4}{x-1}$ geeft $g'(x) = \frac{(x-1) \cdot 0 - 4 \cdot 1}{(x-1)^2} = \frac{-4}{(x-1)^2}$

$f'(x) + 2 \cdot g'(x) = 0$ geeft $\frac{4}{(x+1)^2} + 2 \cdot \frac{-4}{(x-1)^2} = 0$

$$\frac{4}{(x+1)^2} = \frac{8}{(x-1)^2}$$

$$\frac{1}{(x+1)^2} = \frac{2}{(x-1)^2}$$

$$2(x+1)^2 = (x-1)^2$$

$$2(x^2 + 2x + 1) = x^2 - 2x + 1$$

$$2x^2 + 4x + 2 = x^2 - 2x + 1$$

$$x^2 + 6x + 1 = 0$$

$$D = 6^2 - 4 \cdot 1 \cdot 1 = 32, \text{ dus } \sqrt{D} = 4\sqrt{2}$$

$$x = \frac{-6 \pm 4\sqrt{2}}{2} = -3 \pm 2\sqrt{2} \vee x = -3 - 2\sqrt{2}$$

vold. vold.

65 a $p = \frac{3x}{x+5}$ substitueren in $N = \frac{4p-1}{2p+3}$ geeft

$$N = \frac{4 \cdot \frac{3x}{x+5} - 1}{2 \cdot \frac{3x}{x+5} + 3} = \frac{\left(\frac{12x}{x+5} - 1\right) \cdot (x+5)}{\left(\frac{6x}{x+5} + 3\right) \cdot (x+5)} = \frac{12x - (x+5)}{6x + 3(x+5)} = \frac{12x - x - 5}{6x + 3x + 15} = \frac{11x - 5}{9x + 15} \text{ mits } x \neq -5$$

b $N = 9$ geeft $\frac{11x-5}{9x+15} = 9$
 $11x - 5 = 9(9x + 15)$
 $11x - 5 = 81x + 135$
 $-70x = 140$
 $x = -2$

Bladzijde 172

66 a tijd heenreis = $\frac{d}{12}$ uur en tijd terugreis = $\frac{d}{48}$ uur, dus $t = \frac{d}{12} + \frac{d}{48}$.

b gemiddelde snelheid = $\frac{\text{afstand}}{\text{tijd}}$ en afstand = $2d$, dus $\bar{v} = \frac{2d}{\frac{d}{12} + \frac{d}{48}}$.

Teller en noemer delen door d geeft $\bar{v} = \frac{2}{\frac{1}{12} + \frac{1}{48}}$.

De gemiddelde snelheid is $\bar{v} = \frac{2}{\frac{1}{12} + \frac{1}{48}} = 19,2 \text{ km/uur}$.

c $h = \frac{2}{\frac{1}{a} + \frac{1}{b}} \cdot \frac{ab}{ab} = \frac{2ab}{b+a} = \frac{2ab}{a+b}$

d $h = \frac{3}{\frac{1}{a} + \frac{1}{2a} + \frac{1}{3a}} \cdot \frac{6a}{6a} = \frac{18a}{6+3+2} = \frac{18a}{11} = \frac{18}{11} \cdot a$, dus $p = 1\frac{7}{11}$.

Bladzijde 173

67 $y = \frac{2}{x}$
 $xy = 2$
 $x = \frac{2}{y}$

68 $x = 3$ invullen in $x = \frac{3y}{y-1}$ geeft $3 = \frac{3y}{y-1}$
 $3(y-1) = 3y$
 $3y - 3 = 3y$
 $0y = 3$

Er is geen y die aan deze vergelijking voldoet.

Dus in de formule $x = \frac{3y}{y-1}$ is $x = 3$ niet mogelijk.

Bladzijde 174

69 a $y = \frac{x}{x+2}$
 $y(x+2) = x$
 $xy + 2y = x$
 $xy - x = -2y$
 $x(y-1) = -2y$
 $x = \frac{-2y}{y-1}$

b $y = \frac{x-5}{x}$
 $xy = x-5$
 $xy - x = -5$
 $x(y-1) = -5$
 $x = \frac{-5}{y-1}$

70 a $y = \frac{3x+2}{x-4}$
 $y(x-4) = 3x+2$
 $xy - 4y = 3x+2$
 $xy - 3x = 4y+2$
 $x(y-3) = 4y+2$
 $x = \frac{4y+2}{y-3}$

b $y = 20 - \frac{15}{x+1}$
 $y(x+1) = 20(x+1) - 15$
 $xy + y = 20x + 20 - 15$
 $xy - 20x = 5 - y$
 $x(y-20) = 5 - y$
 $x = \frac{5-y}{y-20}$

71 a $y = \frac{ax+4}{x+1}$
 $y(x+1) = ax+4$
 $xy + y = ax+4$
 $xy - ax = 4 - y$
 $x(y-a) = 4 - y$
 $x = \frac{4-y}{y-a}$
 $x = \frac{1}{y-3} - 1 = \frac{1}{y-3} - \frac{y-3}{y-3} = \frac{4-y}{y-3}$
 Dus $a = 3$.

b $y = 2 - \frac{3x+b}{x-1}$
 $y(x-1) = 2(x-1) - (3x+b)$
 $xy - y = 2x - 2 - 3x - b$
 $xy + x = y - 2 - b$
 $x(y+1) = y - 2 - b$
 $x = \frac{y-2-b}{y+1}$

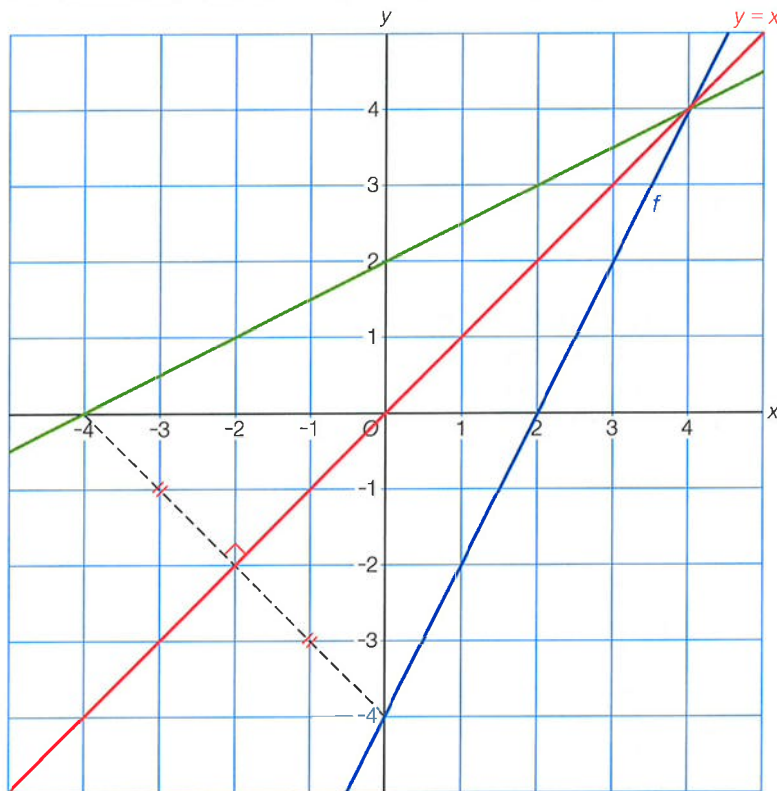
Er moet gelden $y - 2 - b = y - 3$, dus $b = 1$.

c $y = \frac{x-2}{x-1}$
 $y(x-1) = x-2$
 $xy - y = x-2$
 $xy - x = y-2$
 $x(y-1) = y-2$
 $x = \frac{y-2}{y-1}$

d $y = 320 - \frac{18}{x-1}$
 $y(x-1) = 320(x-1) - 18$
 $xy - y = 320x - 320 - 18$
 $xy - 320x = y - 338$
 $x(y-320) = y - 338$
 $x = \frac{y-338}{y-320}$

c $y = \frac{1}{x} + \frac{3}{2x} + 4$
 $y - 4 = \frac{2}{2x} + \frac{3}{2x}$
 $y - 4 = \frac{5}{2x}$
 $2x(y-4) = 5$
 $2x = \frac{5}{y-4}$
 $x = \frac{5}{2(y-4)}$

- 72 a,b** De lijn $y = x$ gaat door de punten $(0, 0)$ en $(4, 4)$.
De grafiek van f gaat door de punten $(0, -4)$ en $(2, 0)$.



- b** Spiegelen van $(0, -4)$ geeft $(-4, 0)$.
Spiegelen van $(2, 0)$ geeft $(0, 2)$.
- c** Stel $y = ax + b$ met $a = \frac{\Delta y}{\Delta x} = \frac{2 - 0}{0 - -4} = \frac{1}{2}$.
- $$\left. \begin{array}{l} y = \frac{1}{2}x + b \\ (0, 2) \end{array} \right\} b = 2$$
- De formule is $y = \frac{1}{2}x + 2$.

Bladzijde 176

- 73 a** Voor f geldt $y = \frac{2x}{x+3}$, dus voor f^{inv} geldt $x = \frac{2y}{y+3}$.

$$\begin{aligned} x = \frac{2y}{y+3} \text{ geeft } x(y+3) &= 2y \\ xy + 3x &= 2y \\ xy - 2y &= -3x \\ y(x-2) &= -3x \\ y &= \frac{-3x}{x-2} \\ y &= \frac{3x}{2-x} \end{aligned}$$

$$\text{Dus } f^{\text{inv}}(x) = \frac{3x}{2-x}.$$

b Voor f geldt $y = \frac{3x-1}{4x+5}$, dus voor f^{inv} geldt $x = \frac{3y-1}{4y+5}$.

$$\begin{aligned} x = \frac{3y-1}{4y+5} \text{ geeft } x(4y+5) &= 3y-1 \\ 4xy+5x &= 3y-1 \\ 4xy-3y &= -5x-1 \\ y(4x-3) &= -5x-1 \\ y &= \frac{-5x-1}{4x-3} \\ y &= \frac{5x+1}{3-4x} \end{aligned}$$

Dus $f^{\text{inv}}(x) = \frac{5x+1}{3-4x}$.

c Voor f geldt $y = \frac{ax+b}{cx+d}$, dus voor f^{inv} geldt $x = \frac{ay+b}{cy+d}$.

$$\begin{aligned} x = \frac{ay+b}{cy+d} \text{ geeft } x(cy+d) &= ay+b \\ cxy+dx &= ay+b \\ cxy-ay &= -dx+b \\ y(cx-a) &= -dx+b \\ y &= \frac{-dx+b}{cx-a} \end{aligned}$$

Dus $f^{\text{inv}}(x) = \frac{-dx+b}{cx-a}$.

74 a Voor f geldt $y = 3 - \frac{4}{x+2}$, dus voor f^{inv} geldt $x = 3 - \frac{4}{y+2}$.

$$\begin{aligned} x = 3 - \frac{4}{y+2} \text{ geeft } x(y+2) &= 3(y+2) - 4 \\ xy+2x &= 3y+6-4 \\ xy-3y &= 2-2x \\ y(x-3) &= 2-2x \\ y &= \frac{2-2x}{x-3} \end{aligned}$$

Dus $f^{\text{inv}}(x) = \frac{2-2x}{x-3}$ en er geldt inderdaad dat $g(x) = f^{\text{inv}}(x)$.

b Voor h geldt $y = \frac{5-x}{x-6}$, dus voor h^{inv} geldt $x = \frac{5-y}{y-6}$.

$$\begin{aligned} x = \frac{5-y}{y-6} \text{ geeft } x(y-6) &= 5-y \\ xy-6x &= 5-y \\ xy+y &= 6x+5 \\ y(x+1) &= 6x+5 \\ y &= \frac{6x+5}{x+1} \end{aligned}$$

Dus $h^{\text{inv}}(x) = \frac{6x+5}{x+1}$.

$$k(x) = 5 + \frac{x}{x+1} = \frac{5(x+1)}{x+1} + \frac{x}{x+1} = \frac{5x+5+x}{x+1} = \frac{6x+5}{x+1}$$

Dus er geldt inderdaad dat $k(x) = h^{\text{inv}}(x)$.

Bladzijde 177

75 Als (p, q) op de grafiek van f ligt, dan ligt (q, p) op de grafiek van f^{inv} .

En als (q, p) ook op de grafiek van f ligt, dan is (q, p) dus een snijpunt van de grafieken van f en f^{inv} .

Als (q, p) op de grafiek van f^{inv} ligt, dan ligt (p, q) op de grafiek van f^{inv} .

En als (p, q) ook op de grafiek van f ligt, dan is (p, q) dus een snijpunt van de grafieken van f en f^{inv} .

Dus bewering 1 is waar, en dus is bewering 2 niet waar.

76 Voor f geldt $y = \frac{2x+1}{3x+4}$, dus voor f^{inv} geldt $x = \frac{2y+1}{3y+4}$.

$$\begin{aligned} x = \frac{2y+1}{3y+4} \text{ geeft } x(3y+4) &= 2y+1 \\ 3xy+4x &= 2y+1 \\ 3xy-2y &= 1-4x \\ y(3x-2) &= 1-4x \\ y &= \frac{1-4x}{3x-2} \end{aligned}$$

Dus $f^{\text{inv}}(x) = \frac{1-4x}{3x-2}$.

$$\begin{aligned} f(x) = f^{\text{inv}}(x) \text{ geeft } \frac{2x+1}{3x+4} &= \frac{1-4x}{3x-2} \\ (2x+1)(3x-2) &= (3x+4)(1-4x) \\ 6x^2-4x+3x-2 &= 3x-12x^2+4-16x \\ 18x^2+12x-6 &= 0 \\ 3x^2+2x-1 &= 0 \\ D = 2^2-4 \cdot 3 \cdot -1 &= 16 \\ x = \frac{-2+4}{6} = \frac{1}{3} \vee x = \frac{-2-4}{6} &= -1 \\ \text{vold.} \quad \quad \quad \text{vold.} \end{aligned}$$

$f(-1) = \frac{-2+1}{-3+4} = -1$, dus $A(-1, -1)$.

$f(\frac{1}{3}) = \frac{\frac{2}{3}+1}{1+4} = \frac{1}{3}$, dus $B(\frac{1}{3}, \frac{1}{3})$.

Opmerking

Je kunt deze opgave ook maken door de vergelijking $f(x) = x$ op te lossen. Je vindt dan twee oplossingen. Omdat is gegeven dat de grafieken van f en f^{inv} elkaar snijden in twee punten, zijn deze oplossingen de x -coördinaten van deze snijpunten.

77 a Voor f geldt $y = \frac{ax-2}{2x-5}$, dus voor f^{inv} geldt $x = \frac{ay-2}{2y-5}$.

$$\begin{aligned} x = \frac{ay-2}{2y-5} \text{ geeft } x(2y-5) &= ay-2 \\ 2xy-5x &= ay-2 \\ 2xy-ay &= 5x-2 \\ y(2x-a) &= 5x-2 \\ y &= \frac{5x-2}{2x-a} \end{aligned}$$

Dus $f^{\text{inv}}(x) = \frac{5x-2}{2x-a}$.

$$g(x) = 2\frac{1}{2} + \frac{4}{x-2} = \frac{5}{2} + \frac{4}{x-2} = \frac{5(x-2)}{2(x-2)} + \frac{8}{2(x-2)} = \frac{5x-10+8}{2(x-2)} = \frac{5x-2}{2x-4}$$

Er moet gelden $2x-a = 2x-4$, dus $a = 4$.

- b** Als de grafieken van f en f^{inv} geen gemeenschappelijke punten hebben op de lijn $y = x$, dan heeft de vergelijking $f(x) = x$ geen oplossingen.

$f(x) = x$ geeft $\frac{ax-2}{2x-5} = x$

$$\begin{aligned} ax-2 &= x(2x-5) \\ ax-2 &= 2x^2-5x \\ 2x^2-ax-5x+2 &= 0 \\ 2x^2-(a+5)x+2 &= 0 \\ D &= (-(a+5))^2-4 \cdot 2 \cdot 2 = (a+5)^2-16 \end{aligned}$$

Geen oplossingen voor $D < 0$, dus voor $(a+5)^2-16 < 0$

$$(a+5)^2 < 16$$

$$-9 < a < -1$$

De grafieken van f en f^{inv} hebben geen gemeenschappelijke punten op de lijn $y = x$ voor $-9 < a < -1$.

78 Voor f geldt $y = \frac{ax+b}{x+1}$, dus voor f^{inv} geldt $x = \frac{ay+b}{y+1}$.

$$\begin{aligned} x = \frac{ay+b}{y+1} \text{ geeft } x(y+1) &= ay+b \\ xy+x &= ay+b \\ xy-ay &= b-x \\ y(x-a) &= b-x \\ y &= \frac{b-x}{x-a} \end{aligned}$$

Dus $f^{\text{inv}}(x) = \frac{b-x}{x-a}$.

$$\begin{aligned} f(3) = f^{\text{inv}}(3) \text{ geeft } \frac{3a+b}{4} &= \frac{b-3}{3-a} \\ (3a+b)(3-a) &= 4(b-3) \\ 9a-3a^2+3b-ab &= 4b-12 \\ -ab-b &= 3a^2-9a-12 \\ b(-a-1) &= 3a^2-9a-12 \\ b &= \frac{3a^2-9a-12}{-a-1} \end{aligned}$$

$$\begin{aligned} f(5) = f^{\text{inv}}(5) \text{ geeft } \frac{5a+b}{6} &= \frac{b-5}{5-a} \\ (5a+b)(5-a) &= 6(b-5) \\ 25a-5a^2+5b-ab &= 6b-30 \\ -ab-b &= 5a^2-25a-30 \\ b(-a-1) &= 5a^2-25a-30 \\ b &= \frac{5a^2-25a-30}{-a-1} \end{aligned}$$

$$\frac{3a^2-9a-12}{-a-1} = \frac{5a^2-25a-30}{-a-1}$$

$$3a^2-9a-12 = 5a^2-25a-30$$

$$-2a^2+16a+18=0$$

$$a^2-8a-9=0$$

$$(a-9)(a+1)=0$$

$$a=9 \vee a=-1$$

vold. vold. niet

$$a=9 \text{ geeft } b = \frac{3 \cdot 9^2 - 9 \cdot 9 - 12}{-9-1} = -15$$

Dus voor $a=9$ en $b=-15$ snijden de grafieken van f en f^{inv} elkaar in de punten A en B met $x_A=3$ en $x_B=5$.

Opmerking

Omdat is gegeven dat de grafieken van f en f^{inv} elkaar snijden in twee punten, mag je in deze opgave ook uitgaan van $f(x_A)=x_A$ en $f(x_B)=x_B$, oftewel $f(3)=3$ en $f(5)=5$.

Diagnostische toets

Bladzijde 180

1 a
$$\begin{cases} 2x-3y=7 \\ 4x+3y=23 \end{cases} + \frac{6x}{6x} = 30 + \begin{cases} x=5 \\ 2x-3y=7 \end{cases} \Rightarrow \begin{cases} 10-3y=7 \\ -3y=-3 \\ y=1 \end{cases}$$

De oplossing is $(x, y) = (5, 1)$.

$$\text{b } \begin{cases} 4x + 5y = 27 \\ -2x + 3y = 25 \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} 4x + 5y = 27 \\ -4x + 6y = 50 \end{cases} +$$

$$\frac{11y = 77}{y = 7} +$$

$$\begin{cases} 4x + 5y = 27 \\ 4x + 5y = 27 \end{cases} \begin{cases} 4x + 35 = 27 \\ 4x = -8 \\ x = -2 \end{cases}$$

De oplossing is $(x, y) = (-2, 7)$.

$$\text{c } \begin{cases} 2x + 3y = 7 \\ 5x - 2y = 8 \end{cases} \begin{vmatrix} 2 \\ 3 \end{vmatrix} \text{ geeft } \begin{cases} 4x + 6y = 14 \\ 15x - 6y = 24 \end{cases} +$$

$$\frac{19x = 38}{x = 2} +$$

$$\begin{cases} 2x + 3y = 7 \\ 2x + 3y = 7 \end{cases} \begin{cases} 4 + 3y = 7 \\ 3y = 3 \\ y = 1 \end{cases}$$

De oplossing is $(x, y) = (2, 1)$.

$$\text{2 } \begin{cases} 3x - 2y = 14 \\ 2x + 5y = 3 \end{cases} \begin{vmatrix} 2 \\ 3 \end{vmatrix} \text{ geeft } \begin{cases} 6x - 4y = 28 \\ 6x + 15y = 9 \end{cases} -$$

$$\frac{-19y = 19}{y = -1} -$$

$$\begin{cases} 2x - 5 = 3 \\ 2x = 8 \\ x = 4 \end{cases}$$

Dus $S(4, -1)$.

$$\text{3 } (-6, 18) \text{ op } y = ax^2 + bx \text{ geeft } 36a - 6b = 18 \text{ oftewel } 6a - b = 3.$$

$$(30, -450) \text{ op } y = ax^2 + bx \text{ geeft } 900a + 30b = -450 \text{ oftewel } 30a + b = -15.$$

$$\begin{cases} 6a - b = 3 \\ 30a + b = -15 \end{cases} +$$

$$\frac{36a = -12}{a = -\frac{1}{3}} +$$

$$\begin{cases} -10 + b = -15 \\ b = -5 \end{cases}$$

De formule is $y = -\frac{1}{3}x^2 - 5x$.

$$\text{4 a } 3x - y = 7 \text{ oftewel } y = 3x - 7$$

Substitutie van $y = 3x - 7$ in $(x - 4)^2 + y^2 = 5$ geeft $(x - 4)^2 + (3x - 7)^2 = 5$

$$x^2 - 8x + 16 + 9x^2 - 42x + 49 = 5$$

$$10x^2 - 50x + 60 = 0$$

$$x^2 - 5x + 6 = 0$$

$$(x - 2)(x - 3) = 0$$

$$x = 2 \vee x = 3$$

$x = 2$ invullen in $y = 3x - 7$ geeft $y = 6 - 7 = -1$.

$x = 3$ invullen in $y = 3x - 7$ geeft $y = 9 - 7 = 2$.

Dus $(x, y) = (2, -1) \vee (x, y) = (3, 2)$.

$$\text{b } x + 4y = 2 \text{ oftewel } x = 2 - 4y$$

Substitutie van $x = 2 - 4y$ in $xy = -6$ geeft $(2 - 4y)y = -6$

$$2y - 4y^2 = -6$$

$$-4y^2 + 2y + 6 = 0$$

$$2y^2 - y - 3 = 0$$

$$D = (-1)^2 - 4 \cdot 2 \cdot -3 = 25$$

$$y = \frac{1 \pm 5}{4} = 1\frac{1}{2} \vee y = \frac{1 - 5}{4} = -1$$

$y = 1\frac{1}{2}$ invullen in $x = 2 - 4y$ geeft $x = 2 - 6 = -4$.

$y = -1$ invullen in $x = 2 - 4y$ geeft $x = 2 + 4 = 6$.

Dus $(x, y) = (-4, 1\frac{1}{2}) \vee (x, y) = (6, -1)$.

5 a $3x^3 + 5 = 86$

$$3x^3 = 81$$

$$x^3 = 27$$

$$x = 3$$

b $5x^4 - 6 = 9$

$$5x^4 = 15$$

$$x^4 = 3$$

$$x = \sqrt[4]{3} \vee x = -\sqrt[4]{3}$$

c $2x^3 + 19 = 5$

$$2x^3 = -14$$

$$x^3 = -7$$

$$x = \sqrt[3]{-7}$$

d $\frac{1}{2}(x+2)^4 = \frac{1}{32}$

$$(x+2)^4 = \frac{1}{16}$$

$$x+2 = \frac{1}{2} \vee x+2 = -\frac{1}{2}$$

$$x = -1\frac{1}{2} \vee x = -2\frac{1}{2}$$

e $100 - (2x+1)^5 = 68$

$$-(2x+1)^5 = -32$$

$$(2x+1)^5 = 32$$

$$2x+1 = 2$$

$$2x = 1$$

$$x = \frac{1}{2}$$

f $(2x+4)^3 = 10$

$$2x+4 = \sqrt[3]{10}$$

$$2x = -4 + \sqrt[3]{10}$$

$$x = -2 + \frac{1}{2} \cdot \sqrt[3]{10}$$

6 a $x^3 = x^2 + 20x$

$$x^3 - x^2 - 20x = 0$$

$$x(x^2 - x - 20) = 0$$

$$x(x+4)(x-5) = 0$$

$$x = 0 \vee x = -4 \vee x = 5$$

b $x^4 - 6x^2 + 5 = 0$

$$\text{Stel } x^2 = u.$$

$$u^2 - 6u + 5 = 0$$

$$(u-1)(u-5) = 0$$

$$u = 1 \vee u = 5$$

$$x^2 = 1 \vee x^2 = 5$$

$$x = 1 \vee x = -1 \vee x = \sqrt{5} \vee x = -\sqrt{5}$$

c $x^8 + x^4 = 42$

$$x^8 + x^4 - 42 = 0$$

$$\text{Stel } x^4 = u.$$

$$u^2 + u - 42 = 0$$

$$(u-6)(u+7) = 0$$

$$u = 6 \vee u = -7$$

$$x^4 = 6 \vee x^4 = -7$$

$$x = \sqrt[4]{6} \vee x = -\sqrt[4]{6}$$

d $5x^4 + 1 = 6x^2$

$$5x^4 - 6x^2 + 1 = 0$$

$$\text{Stel } x^2 = u.$$

$$5u^2 - 6u + 1 = 0$$

$$D = (-6)^2 - 4 \cdot 5 \cdot 1 = 16$$

$$u = \frac{6+4}{10} = 1 \vee u = \frac{6-4}{10} = \frac{1}{5}$$

$$x^2 = 1 \vee x^2 = \frac{1}{5}$$

$$x = 1 \vee x = -1 \vee x = \sqrt{\frac{1}{5}} \vee x = -\sqrt{\frac{1}{5}}$$

7 a $|2x^3 - 5| = 49$

$$2x^3 - 5 = 49 \vee 2x^3 - 5 = -49$$

$$2x^3 = 54 \vee 2x^3 = -44$$

$$x^3 = 27 \vee x^3 = -22$$

$$x = 3 \vee x = \sqrt[3]{-22}$$

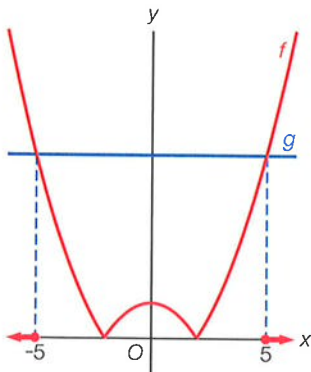
b $f(x) = |x^2 - 4|$ en $g(x) = 21$

$$f(x) = g(x) \text{ geeft } |x^2 - 4| = 21$$

$$x^2 - 4 = 21 \vee x^2 - 4 = -21$$

$$x^2 = 25 \vee x^2 = -17$$

$$x = 5 \vee x = -5$$



$$|x^2 - 4| \geq 21 \text{ geeft } x \leq -5 \vee x \geq 5$$

8 a $(x^2 - 6)(x^2 - 2x) = 0$
 $x^2 - 6 = 0 \vee x^2 - 2x = 0$
 $x^2 = 6 \vee x(x - 2) = 0$
 $x = \sqrt{6} \vee x = -\sqrt{6} \vee x = 0 \vee x = 2$

b $(2x^2 - 1)^2 = (6x + 1)^2$
 $2x^2 - 1 = 6x + 1 \quad \vee 2x^2 - 1 = -6x - 1$
 $2x^2 - 6x - 2 = 0 \quad \vee 2x^2 + 6x = 0$
 $x^2 - 3x - 1 = 0 \quad \vee 2x(x + 3) = 0$
 $D = (-3)^2 - 4 \cdot 1 \cdot -1 = 13$
 $x = \frac{3 + \sqrt{13}}{2} \vee x = \frac{3 - \sqrt{13}}{2} \quad \vee x = 0 \vee x = -3$
 $x = 1\frac{1}{2} + \frac{1}{2}\sqrt{13} \vee x = 1\frac{1}{2} - \frac{1}{2}\sqrt{13} \vee x = 0 \vee x = -3$

c $x(x^2 - 1) = 4(x^2 - 1)$
 $x^2 - 1 = 0 \vee x = 4$
 $x^2 = 1 \vee x = 4$
 $x = 1 \vee x = -1 \vee x = 4$

d $(x^3 - 9x)(x^2 - 3) = x^3 - 9x$
 $x^3 - 9x = 0 \vee x^2 - 3 = 1$
 $x(x^2 - 9) = 0 \vee x^2 = 4$
 $x = 0 \vee x^2 = 9 \vee x = 2 \vee x = -2$
 $x = 0 \vee x = 3 \vee x = -3 \vee x = 2 \vee x = -2$

9 a $\sqrt{3x+5} + 1 = 5$
 $\sqrt{3x+5} = 4$
 $3x + 5 = 16$
 $3x = 11$
 $x = 3\frac{2}{3}$

b $3x = 5\sqrt{x+4}$
kwadrateren geeft
 $9x^2 = 25(x+4)$
 $9x^2 = 25x + 100$
 $9x^2 - 25x - 100 = 0$
 $D = (-25)^2 - 4 \cdot 9 \cdot -100 = 4225$
 $x = \frac{25 + 65}{18} = 5 \vee x = \frac{25 - 65}{18} = -2\frac{2}{9}$
 $x = 5$ geeft $15 = 5\sqrt{5+4}$ vold.
 $x = -2\frac{2}{9}$ geeft $-6\frac{2}{3} = 5\sqrt{-2\frac{2}{9}+4}$ vold. niet

c $x = \sqrt{x+6}$
 $x - 6 = \sqrt{x}$
kwadrateren geeft
 $(x-6)^2 = x$
 $x^2 - 12x + 36 = x$
 $x^2 - 13x + 36 = 0$
 $(x-4)(x-9) = 0$
 $x = 4 \vee x = 9$
 $x = 4$ geeft $4 = \sqrt{4+6}$ vold. niet
 $x = 9$ geeft $9 = \sqrt{9+6}$ vold.

d $2x + 3\sqrt{x} = 2$
 $3\sqrt{x} = 2 - 2x$
kwadrateren geeft
 $9x = (2 - 2x)^2$
 $9x = 4 - 8x + 4x^2$
 $-4x^2 + 17x - 4 = 0$
 $D = 17^2 - 4 \cdot -4 \cdot -4 = 225$
 $x = \frac{-17 + 15}{-8} = \frac{1}{4} \vee x = \frac{-17 - 15}{-8} = 4$
 $x = \frac{1}{4}$ geeft $\frac{1}{2} + 3\sqrt{\frac{1}{4}} = 2$ vold.
 $x = 4$ geeft $8 + 3\sqrt{4} = 2$ vold. niet

Bladzijde 181

10 a $\frac{x^2 - 5x + 6}{2x + 4} = 0$
 $x^2 - 5x + 6 = 0$
 $(x-2)(x-3) = 0$
 $x = 2 \vee x = 3$
vold. vold.

b $\frac{x^2 - 4}{2x + 1} = \frac{x^2 - 4}{x - 4}$
 $x^2 - 4 = 0 \vee 2x + 1 = x - 4$
 $x^2 = 4 \vee x = -5$
 $x = 2 \vee x = -2 \vee x = -5$
vold. vold. vold.

$$\text{c } \frac{2x-1}{x+1} = \frac{4x+1}{5x-1}$$

$$\begin{aligned}(2x-1)(5x-1) &= (x+1)(4x+1) \\ 10x^2 - 2x - 5x + 1 &= 4x^2 + x + 4x + 1 \\ 6x^2 - 12x &= 0 \\ 6x(x-2) &= 0 \\ x &= 0 \vee x = 2 \\ \text{vold.} \quad \text{vold.}\end{aligned}$$

$$\text{d } \frac{2x^2-4}{x+5} = 1\frac{3}{4}$$

$$\begin{aligned}\frac{2x^2-4}{x+5} &= \frac{7}{4} \\ 4(2x^2-4) &= 7(x+5) \\ 8x^2 - 16 &= 7x + 35 \\ 8x^2 - 7x - 51 &= 0 \\ D &= (-7)^2 - 4 \cdot 8 \cdot -51 = 1681 \\ x &= \frac{7+41}{16} = 3 \vee x = \frac{7-41}{16} = -2\frac{1}{8} \\ \text{vold.} \quad \text{vold.}\end{aligned}$$

$$\text{11 a } \frac{x^4-16}{x^3-4x} = \frac{(x^2+4)(x^2-4)}{x(x^2-4)} = \frac{x^2+4}{x} \text{ mits } x \neq 2 \wedge x \neq -2$$

$$\text{b } \frac{x^4-5x^2+4}{x^4-1} = \frac{(x^2-1)(x^2-4)}{(x^2+1)(x^2-1)} = \frac{x^2-4}{x^2+1} \text{ mits } x \neq 1 \wedge x \neq -1$$

$$\text{12 a } y = 2x - \frac{x-1}{x-2} = \frac{2x(x-2)}{x-2} - \frac{x-1}{x-2} = \frac{2x^2-4x-x+1}{x-2} = \frac{2x^2-5x+1}{x-2}$$

$$\text{b } y = \frac{3}{x} \left(2 - \frac{x}{x-1} \right) = \frac{3}{x} \left(\frac{2(x-1)}{x-1} - \frac{x}{x-1} \right) = \frac{3}{x} \left(\frac{2x-2-x}{x-1} \right) = \frac{3}{x} \left(\frac{x-2}{x-1} \right) = \frac{3x-6}{x(x-1)}$$

$$\text{c } y = \frac{2x + \frac{x}{x-1}}{x+1} = \frac{\left(2x + \frac{x}{x-1} \right) \cdot (x-1)}{(x+1) \cdot (x-1)} = \frac{2x(x-1) + x}{x^2-1} = \frac{2x^2-2x+x}{x^2-1} = \frac{2x^2-x}{x^2-1}$$

$$\text{d } y = \frac{\frac{2}{x+1} - 3}{4 - \frac{x}{x+1}} = \frac{\left(\frac{2}{x+1} - 3 \right) \cdot (x+1)}{\left(4 - \frac{x}{x+1} \right) \cdot (x+1)} = \frac{2-3(x+1)}{4(x+1)-x} = \frac{2-3x-3}{4x+4-x} = \frac{-3x-1}{3x+4} \text{ mits } x \neq -1$$

$$\text{13 a } N = \frac{4x^2-50}{2x} = 2x - \frac{25}{x}$$

$$\text{b } B = \frac{6p^2-3p+4}{3p} = 2p - 1 + \frac{4}{3p}$$

$$\begin{aligned}\text{14 a } y &= \frac{2x-3}{x+5} \\ y(x+5) &= 2x-3 \\ xy+5y &= 2x-3 \\ xy-2x &= -5y-3 \\ x(y-2) &= -5y-3 \\ x &= \frac{-5y-3}{y-2} \\ x &= \frac{5y+3}{2-y}\end{aligned}$$

$$\begin{aligned}\text{b } y &= 3 + \frac{2x}{x-1} \\ y(x-1) &= 3(x-1) + 2x \\ xy-y &= 3x-3+2x \\ xy-5x &= y-3 \\ x(y-5) &= y-3 \\ x &= \frac{y-3}{y-5}\end{aligned}$$

$$\text{15 a } \text{Voor } f \text{ geldt } y = \frac{2x-1}{5-x}, \text{ dus voor } f^{\text{inv}} \text{ geldt } x = \frac{2y-1}{5-y}.$$

$$\begin{aligned}x = \frac{2y-1}{5-y} \text{ geeft } x(5-y) &= 2y-1 \\ 5x-xy &= 2y-1 \\ -xy-2y &= -5x-1 \\ xy+2y &= 5x+1 \\ y(x+2) &= 5x+1 \\ y &= \frac{5x+1}{x+2}\end{aligned}$$

$$\text{Dus } f^{\text{inv}}(x) = \frac{5x+1}{x+2}.$$

- b** Voor g geldt $y = 2 - \frac{3}{x-4}$, dus voor g^{inv} geldt $x = 2 - \frac{3}{y-4}$.

$$x = 2 - \frac{3}{y-4} \text{ geeft } x(y-4) = 2(y-4) - 3$$

$$xy - 4x = 2y - 8 - 3$$

$$xy - 2y = 4x - 11$$

$$y(x-2) = 4x - 11$$

$$y = \frac{4x-11}{x-2}$$

$$\text{Dus } g^{\text{inv}}(x) = \frac{4x-11}{x-2}.$$